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OPTICAL SPECTRA WITHIN THE ORDINARY X-RAY REGION RECORDED WITH CURVED CRYSTAL

BY

HARALD FLEMBERG

WITH 2 PLATES AND 9 FIGURES IN THE TEXT

COMMUNICATED APRIL 8TH 1942 BY
MANNE SIEGBAHN AND A. E. LINDH

STOCKHOLM
ALMQVIST & WIKSELLS BOKTRYCKERI-A.-B.

BERLIN NW 7	LONDON	PARIS
„NATURA" BUCHHANDLUNG	H. K. LEWIS & CO., LTD	LIBRAIRIE C. KLINCKSIECK
11 KARLSTRASSE	136, GOWER STREET	11 RUE DE LILLE

1942

OPTICAL SPECTRA WITHIN THE ORDINARY X-RAY REGION RECORDED WITH CURVED CRYSTAL

INAUGURAL DISSERTATION

BY

HARALD FLEMBERG
FIL. LIC., SÖDERM.

BY DUE PERMISSION OF THE PHILOSOPHICAL FACULTY OF THE UNIVERSITY OF UPPSALA, TO BE PUBLICLY DISCUSSED IN THE LECTURE HALL OF THE PHYSICAL INSTITUTE, MAY 26TH, 1942, AT 10 A. M., FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UPPSALA 1942
ALMQVIST & WIKSELLS BOKTRYCKERI-A.-B.

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Preface.

This investigation has been carried out at the Physical Institute of Uppsala University. It was started in the autumn of 1937; but no results worth mentioning were obtained until a year later. These earlier attempts to record optical spectra with a curved crystal were carried out with a spectrograph originally intended for ordinary X-ray recordings and only slightly altered for its new purpose. At the end of 1938, a short account of the results then obtained was published in Zs. f. Phys. Vol. 111. At the same time, a new and more suitable spectrograph was constructed. This was ready for use in the spring of 1939. With the new spectrograph, it was possible to record optical spectra of the elements fluorine, magnesium and aluminium in the wave-length region 17—6 Å. U. The results given here are of these later recordings.

I wish to express my deepest gratitude to my teacher, the director of the Physical Institute, Professor Axel E. Lindh, who gave me permission to construct the new spectrograph, at a time when it was still impossible to draw any definite conclusions as to the value of the method. Moreover, he has continually placed all the resources of the Institute at my disposal. The excellent technical equipment, together with the encouraging interest with which he has always followed my work, has made this investigation possible. For all his help in these and many other matters, I am sincerely grateful.

It was my privilege to receive my first introduction to spectroscopic work under the guidance of Professor Manne

Siegbahn, the former director of the Institute. For the valuable experience thereby obtained and for his continuous kindness towards me, I should like to express my warmest thanks.

I am also greatly indebted to Dr. Erik Bäcklin and Dr. Bengt Edlén, who have many times placed their wide experience at my disposal, and who in many other ways have been extremely helpful towards me.

For the translation, I am indebted to Mrs. William Cameron.

Finally, I should like to thank my wife for her continuous encouragement and unflagging interest in my work.

Uppsala, April 1942.

Harald Flemberg.

ARKIV FÖR MATEMATIK, ASTRONOMI OCH FYSIK.
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Contents.

		Page.
I.	Introduction	2
	The conquest of the ultra-violet region	2
	The possibilities of further advances towards shorter wave-lengths	3
II.	The Properties of the Concave Crystal Spectrographs	4
	Derivation of the interpolation formula	5
	Broadening effects	7
	Some sources of error	8
III.	Experimental Arrangements	10
	A. The Spectrograph	10
	Some notes on the construction of the spectrograph	10
	The body of the spectrograph	11
	The spark-chamber — the X-ray tube	12
	The spectroscopic unit	15
	The plate-holder	16
	The crystal-holder and crystal-table	17
	The crystals	19
	B. Remaining Experimental Equipment	20
	The pumps	20
	The high voltage sets	21
	The thermostat	21
IV.	The Recording and Measurement of the Spectrograms	22
	Adjustment of the spectrograph	22
	Plate material	24
	Spark-electrodes and absorbing foils	24
	The reference lines	25
	Method of exposure	26
	The measurement of the spectrograms	27

Page.
V. Experimental Results 27
Wave-length units 27
The results 28
VI. Discussion of the Results 35
Accuracy of the wave-length values 35
Comparison with earlier measurements and theoretical values . . 38
The satellites 44
Summary 45
References 46

I. Introduction.

The conquest of the ultra-violet region.

By about the middle of the last century, ultra-violet spectra containing lines with wave-lengths down to 1850 Å. U. had been successfully obtained. A further step towards shorter wave-lengths met with very great difficulties. Schumann (4, 24), who was the pioneer in this, realized that the absorption in the light-path in the spectrographs then used had to be reduced in every possible way. He therefore suggested the method still used today, that of preparing photographic plates with emulsion containing little gelatine. Furthermore, he replaced the prisms and lenses of quartz which had hitherto been used by an optical system of fluorite. Finally, he eliminated the absorption of the air by enclosing the apparatus in an evacuable receptacle. With such fluorite vacuum spectrographs Schumann had, by about 1900, obtained spectra to as far down as 1250 Å. U., and in isolated advantageous cases even lower. The weakness of this method, however, was that the determinations of the wave-lengths became very inaccurate, the dispersion of fluorite in this region being known only through extrapolation.

The short-wave limit for the Schumann region, 1250 Å. U., was fixed by the absorption of the fluorite. This difficulty was overcome by Lyman (4, 24) who constructed a vacuum spectrograph with concave grating. In this, no absorbing medium need exist in the light-path. The very first experiments were promising; and through successive improvements, he succeeded by 1917 in lowering the short-wave limit to 500 Å. U. The use of concave grating, moreover, allowed of an exact determination of the wave-lengths.

The next advance was due to Millikan and his collaborators Bowen and Sawyer (4, 24). They used special lightly ruled gratings, the reflecting power of which in this extreme short-wave region was superior to that of the older gratings.

Still more important, however, was the fact that they introduced a new light source, the vacuum or hot spark. This type of discharge had already been described by WOOD (35); but it was MILLIKAN who discovered what a rich source of extreme short-wave radiation it was. This is due to the high ionization power of the vacuum spark, because it is of course the highly ionized atoms, the »stripped atoms», that emit the short-wave radiation. With this equipment, they obtained spectra of high intensity in the region from 300 to 200 Å. U., while the short-wave limit was about 150 Å. U. (25).

Both LYMAN and MILLIKAN let the incident radiation strike the grating at or almost at right angles. The reflecting power of the grating in this position is, practically speaking, zero, when the wave-length of the light approaches 100 Å. U. However, if the light be allowed to strike the grating at a very small glancing angle, the reflecting power is retained for the short-wave radiation. Thus COMPTON and DOAN (7) succeeded in 1925 in obtaining diffraction spectra of short-wave X-ray lines using a plane grating and grazing incidence. This arrangement was first used in ultra-violet spectroscopy by HOAG (18), who made use of a concave grating spectrograph where the angle of incidence was 80°. The method was soon frequently applied both in ultra-violet spectroscopy and for recording extreme soft X-ray spectra, the examination of which either cannot be carried out at all with crystals, or only with very great difficulty.

The development of this method was due chiefly to SIEGBAHN (30) and his collaborators. Thus EDLÉN and ERICSON (9, 11) succeeded in 1929 for the first time in obtaining optical spectra all the way down into the extreme soft X-ray region below 100 Å. U. A great many investigations by EDLÉN (8) and others led to the short-wave limit attained for optical spectra being lowered some years later to the neighbourhood of 30 Å. U. Finally, TYRÉN (32, 33, 34) has, during recent years, obtained particularly fine spectra inside this difficult region. He has been able to record the first Lyman line of O VIII with a wave-length of 19.0 Å. U. In the isoelectronic sequence He I, Li II, — he has reached F VIII, the resonance line of which lies at 16.8 Å. U. The shortest wave-length measured was 12.1 Å. U. for a line of Fe XVII.

The possibilities of further advances towards shorter wave-lengths.

TYRÉN (32) used for his investigations a concave grating spectrograph with an angle of incidence of 88° 58′. If we

calculate in the usual way the short-wave limit for the radiation that is totally reflected at this angle, disregarding the absorption, we get 6.9 Å. U. The absorption — which in this region cannot be neglected — causes the absence of a sharp limit and results in a continuous decrease in the reflecting power as the critical wave-length is approached. This is in accordance with the fact that TYRÉN could, only with the greatest difficulty, obtain the X-ray line Si $K\alpha_{1,2}$ of wave-length 7.1 Å. U. If we wish to enlarge to any extent the region in which this method could be used — to get below the present short-wave limit of 12 Å. U. — the angle of incidence must obviously be further enlarged. With concave gratings, this presents great difficulties. With plane gratings, the large angle of incidence is no obstacle; but since it is necessary to work with a very narrow beam, the grating surface is greatly restricted, thus reducing both the intensity and the resolving power.

On the other hand, no obstacles are met with if the interferences in a crystal lattice are used in the usual way for recording X-ray spectra with wave-lengths up to at least 15 Å. U. Furthermore, it may be easily shown that by using a curved crystal and the Johann method, X-ray spectra comparable with TYRÉN's may be obtained inside the wave-length region up to at least 10 Å. U., and after considerably shorter exposures than his.

The present author concluded from this that the method with curved crystal ought to be tried for optical spectra also. A success in these investigations ought, ipso facto, to be of interest as adding the last link between the ultra-violet region and the X-ray region. If it is really the case that a sufficiently intense vacuum spark emits optical radiation of shorter wave-lengths than hitherto recorded, this method ought to be a suitable one for recording them. And, in fact, spark spectra of the elements fluorine, magnesium and aluminium, situated in the wave-length region from 17—6 Å. U., were successfully recorded in this way.

II. The Properties of the Concave Crystal Spectrographs.

The method suggested by JOHANN (22) involves a crystal's being curved so as to form part of the surface of a cylinder. The selective reflection takes place in the concave surface of the crystal against those atom planes that are parallel to the surface of the crystal. As with a concave grating, the spectral

lines are focussed on the surface of a cylinder, the diameter of which corresponds to the radius of curvature of the crystal. Although the properties of spectrographs built according to this principle have been the subject of several investigations, nevertheless we may consider them again. When a vacuum spark is used as the source of radiation instead of an X-ray tube, certain difficulties which have otherwise been solved arise again. It is, therefore, necessary to give a short survey of the most important properties of such a spectrograph with curved crystal.

Derivation of the interpolation formula. It would seem natural to use for the recordings a plate-holder ground in the form of a cylinder of diameter equal to the radius of the cylindrical surface of the crystal-holder. In the present case, however, the diameter of such a plate-holder would be only 500 mm. It would, therefore, be necessary to use film as the photographic material. The disadvantages of this are scarcely compensated by any real advantage. The cylindrical surface of the plate-holder can very seldom be made identical with the focussing cylinder, since the radius of a curved crystal in most cases will not be exactly the same as that of the crystal-holder. It has, therefore, been considered advantageous to record on plane glass plates placed at a tangent to the focussing cylinder. The glancing angles for lines on any given plate differ by a maximum of 2°. For so little an angle range, the distance between the plate and the focussing circle is very small.

One characteristic which all concave crystal methods have in common is that they are only suitable for relative measurements. The wave-lengths of the optical lines have, therefore, been determined by using known X-ray lines as reference lines. The interpolation must be made with the glancing angles. This roundabout way through the glancing angles cannot, unfortunately, be avoided, as the calculations according to Bragg's law must be made with different lattice constants for the different orders.

In the following derivation of the interpolation formula, the deviation from exact focussing is temporarily omitted, and the beams reflected from the mid-point of the crystal are considered as mathematical lines. Fig. 1 shows a section at right angles to the axis O of the focussing cylinder. In the following paragraphs, the plane of the figure is, for simplicity, called the »horizontal plane«. The expression »vertically« thus means »at right angles to this plane». The intersection of the focussing cylinder and the horizontal plane is called the focussing

circle. In the figure, the focussing circle touches the plate AB at C, and the crystal at its mid-point D. Let the angle COD be $2\varphi_0$. In addition to the required line L_x, there are on the plate the two reference lines, L_1 and L_2, with known wave-lengths. Assuming that the lattice constant of the crystal and the corrections for normal and anomalous dispersion are known, the glancing angles, φ_1 and φ_2, of the reference lines may be calculated. Let the glancing angle of the required line be φ, and introduce in addition the letters $\beta = \varphi_1 - \varphi_2$

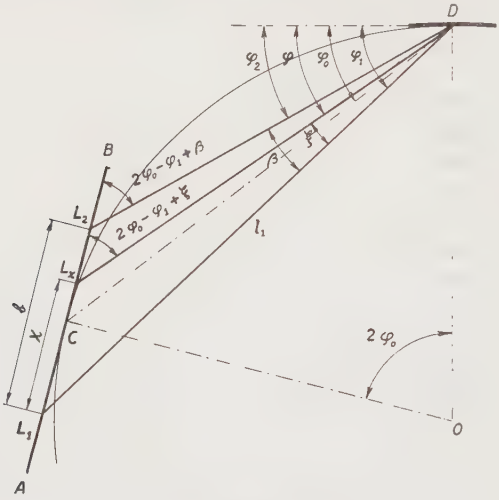

Fig. 1.

and $\xi = \varphi_1 - \varphi$. The distances b and x on the plate are measured accurately in a comparator. Let the distance DL_1 be l_1. From purely geometrical considerations, we obtain the following two equations:

$$\frac{x}{\sin \xi} = \frac{l_1}{\sin (2\varphi_0 - \varphi_1 + \xi)}$$

$$\frac{b}{\sin \beta} = \frac{l_1}{\sin (2\varphi_0 - \varphi_1 + \beta)}$$

Eliminating l_1, and solving the resulting equation for $\tan \xi$, we find:

$$\tan \xi = \frac{x}{b} \tan \beta \frac{1}{1 + \frac{b - x}{b} \tan \beta \cot (2\varphi_0 - \varphi_1)}. \qquad (1)$$

The angle ξ can now be calculated since all the quantities on the right hand side of the equation are known with the exception of φ_0. The angle $2\varphi_0$ can be determined only approximately, the error, however, should never exceed $0°.4$. As φ_0 occurs only in the correction term, such an error will in most cases be insignificant (compare section Accuracy of the wave-length values). The glancing angle φ of the required line may now be obtained from the relation $\varphi = \varphi_1 - \xi$, after which the wave-length of the line can be calculated in the usual way.

Broadening effects. Even if the crystal is perfectly curved, the focussing on the focussing cylinder is not exact. The broadening of the spectral lines, which is a result of the characteristic focussing defect of this method, was estimated by JOHANN (22) as:

$$b = \frac{o^2}{8R} \cot \varphi. \tag{2}$$

The broadening b is measured along the focussing circle, o is the crystal opening in mm, R the radius of curvature of the crystal, and φ the glancing angle of the spectral line. This broadening tends always in the direction of shorter wave-lengths.

The two following factors cause broadening of the spectral lines in the direction of longer wave-lengths. The extent of the source of radiation vertically causes a broadening b' which, in the middle of the spectral lines, is given by

$$b' = \frac{h^2}{2(R \sin \varphi + A) \cos \varphi}. \tag{3}$$

In this formula, which has been derived on the assumption that the plate is a tangent to the focussing cylinder, h is half the height of the source and A the distance from it to the crystal. The letters R and φ denote the same as before.

Since the plate is a tangent, the incident rays strike the plate at an angle equal to the glancing angle φ. Assuming that the radiation penetrates the photographic emulsion, a broadening b'' results, which may be written:

$$b'' = d \cot \varphi \tag{4}$$

where d is the thickness of the photographic emulsion.

If the temperature of the crystal fluctuates during the exposure of a spectrum, a certain variation in the glancing

angles of the spectral lines occurs, which appears on the plate as a broadening of the lines. With a tangential plate, this broadening will be

$$b''' = R\,\alpha \tan \varphi \cdot \Delta t. \tag{5}$$

Here, α is the linear coefficient of thermal expansion of the crystal, at right angles to the reflecting atom planes and Δt the range within which the temperature varies. The other letters denote the same as before. It is only with spectral lines requiring long exposures that the effect of this would be noticable. According to SIEGBAHN (30) pages 47 and 48, α has the values 3.78×10^{-5} for gypsum and 1.53×10^{-5} for mica, which are the crystal lattices used here.

The broadening which occurs when the recording is not made on the focussing cylinder may, with a tangential plate, be neglected, since within the angle range comprised by the plate, the distance between the plate and the focussing cylinder is throughout very small. Besides, any insignificant broadening of this kind does not shift the position of the maximum blackening.

Some sources of error. The broadening effects just dealt with need not necessarily entail any error in the determinations of the wave-lengths. The broadening b in formula (2) is the same for neighbouring lines and does not, therefore, influence the measurements.

The broadening b' in formula (3) may possibly cause a small error, if the active region[1] of the vacuum spark and the focus-spot of the X-ray tube differ to any extent in height.

If the centre of the focus-spot and the middle of the active region inside the spark are at different heights above the horizontal plane, the vertices of the hyperbolic spectral lines are displaced in relation to each other. This usually causes errors in the distances between the lines measured on the plate. For a given difference in height between the focus-spot and the spark, the greatest error occurs when one or other is in the correct position, and the measurements are made along the central horizontal line on the plate (or along two neighbouring lines one on either side of the central line). In this case the error may be found from formula (3), if h be the difference in height between the centre of the focus-spot and the middle of the active region of the spark.

[1] The active region of the spark = that part of the spark which emits the extreme short-wave radiation.

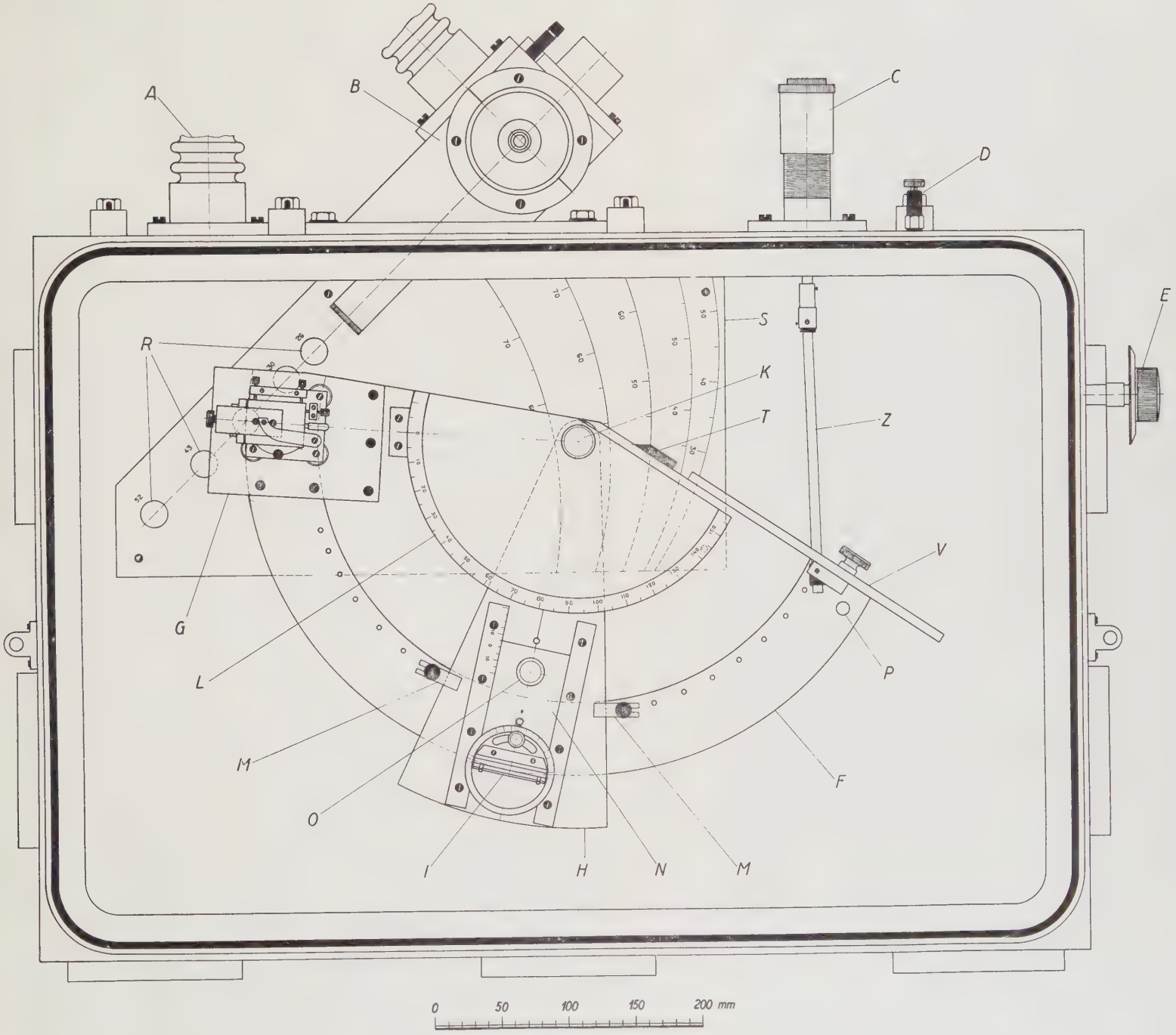

Fig. 2. The spectrograph from above.

The optical lines have considerably longer wave-lengths than the X-ray lines, which have been used as reference lines. The X-ray lines have thus been recorded in higher orders than the optical lines. Since the reference lines have greater penetrating power than the optical lines, the broadening resulting from the oblique incidence on the plate is greater in the former than in the latter. As a result of this, too low wave-length values are obtained for the required optical lines. The maximum error occurs if the reference lines penetrate the photographic emulsion completely, while the optical lines are absorbed on the surface. The maximum error amounts to $\frac{1}{2}b''$, where b'' is calculated from formula (4).

Since the spark-spectrum and the reference-spectrum cannot be exposed at the same time, an error may arise as a result of the mean temperature of the crystal differing for the two exposures. If the mean temperature during the exposure of the reference-spectrum is higher than that during the exposure of the spark-spectrum, the wave-lengths obtained for the optical lines will be too long. The magnitude of the error $|\Delta\lambda|$ in the wave-length of the required line may be written:

$$|\Delta\lambda| = \lambda\alpha|t_2 - t_1|. \qquad (6)$$

In this formula, λ is the wave-length of the line and $t_2 - t_1$ the difference between the mean temperatures during the two exposures. The meaning of α is the same as before.

In the preceding argument we have assumed all the time that the form of the crystal is perfect and constitutes exactly part of the surface of a cylinder. This assumption may, however, in many cases not be justified. Rough errors or irregularities in bending obviously need not be considered, since such crystals would have been rejected as giving unsatisfactory images. On the other hand, it is necessary to take into consideration the fact that the radius of curvature of the crystal may increase or decrease regularly from the mid-point of the crystal towards the edges. Errors arising from an uncritical use of such a crystal for wave-length measurements have been thoroughly discussed by Ingelstam (20). He has also stated how such errors may be avoided. It is only necessary to make sure that the entire crystal opening is used for the recording of both the optical lines and the reference lines. Assuming this, a moderate error in bending produces only a certain fixed broadening of the spectral lines. Since the broadening is the same for all the lines on a given plate, it has no influence on the wave-length measurements.

If the source of radiation lies on the focussing circle, the entire crystal opening is sure to be used. But this arrangement has some drawbacks. Among them, the fact that the angle range illuminated becomes very small, even when the source of radiation is comparatively extensive, and further, the risk that the intensity distribution of the source may be directly imaged upon the plate. In exposures of ordinary X-rays, it is customary, therefore, for the X-ray tube to be placed considerably nearer the crystal. A rather large angle range is thus obtained, within which all the spectral lines are reflected from the entire crystal opening. The only condition necessary for this is that the focus-spot be sufficiently large.

When the source of radiation is a vacuum spark, none of these arrangements is convenient as it stands. The visible source is comparatively small, and the active region is perhaps much smaller still. If the spark be placed on the focussing circle, the angle range illuminated becomes very small. As the position of the spark is not wholly determined by the placing of the electrodes, it is in any case uncertain whether an intended line is illuminated or not. If the spark be placed nearer to the crystal, on the other hand, there is a risk of error in the wave-length measurements. The difficulties disappear if the source of radiation be continually displaced during the exposure so that the glancing angle of the beam incident upon the crystal varies. The method has been sometimes used previously when recording ordinary X-rays. When the source of radiation is a vacuum spark, this method is probably the only practicable one. The distance of the spark from the crystal is now chiefly determined by purely technical considerations.

III. Experimental Arrangements.

A. The Spectrograph.

Some notes on the construction of the spectrograph. The first experiments to record optical spectra with a curved crystal were carried out with a spectrograph designed for the spectroscopy of ordinary X-rays, and only insignificantly changed to suit its new purpose (13). The crystal, the photographic film and the spark were placed in a common vacuum tank, through the lid and bottom of which the spark-electrodes were conveyed. This construction proved utterly unsuitable, as the films were soon blackened from small sparks between the different parts within the spectrograph and from discharges in

the residual gases. *The spark-chamber must, therefore, be placed outside the receptacle of the spectrograph.* The fact that in this case a window may easily be inserted rendering it possible to observe the spark from without, is also an advantage.

The spark-chamber must also be constructed in such a way as to be easily converted into an X-ray tube. The final adjustment of the spectrograph may most conveniently be carried out by the application of X-ray lines, which require only short exposures. X-ray lines must likewise be used as reference lines.

The reasons why the spark ought, during the exposures, to be displaced in relation to the crystal and the plate have already been thoroughly considered. In practice, this is most easily performed if *the crystal and the plate are mutually rigid and their position relative to the fixed spark changed during the given exposure.*

A fairly small radius of curvature ought to be chosen for the crystal. A large radius introduces an unnecessarily great linear dispersion and a corresponding loss in intensity. On the other hand, a small radius of curvature renders impossible the use of crystals that are apt to crack during bending.

All these factors were taken into consideration by the author when designing the spectrograph used for this investigation. Since it is impossible to determine beforehand the wave-length region for which it will be used, the spectrograph was constructed so that the recording arrangements could be varied within certain limits. The spectrograph — except the crystal-holder — was constructed in the workshop at the Institute.

The body of the spectrograph. The vacuum tank of the spectrograph is shaped like a box with internal measurements $720 \times 470 \times 94$ mm^3. The lid and the bottom are identical, and made of iron plates, 20 mm thick. Both plates are strongly reinforced in order to withstand the force of the evacuation. The walls of the box are made up of bars of wrought-iron, with rectangular transverse measurements of 94×33 mm^2, which are planed so as to form a ⌊-shaped cross-section, and are then welded together into a rectangular frame. The upper and lower sides of the frame and the insides of the lid and the bottom are planed level. The frame is pressed firmly to the bottom by means of screw clamps. Its position is then fixed by two pins. The vacuum tightening is achieved by means of rubber bands of circular cross-section, which fit into grooves cut in the two plane surfaces of the frame. All other joints, which require to be vacuum-tight, are made on the same princ-

iple. Three movable feet render it possible to adjust the plane bottom horizontally.

Fig. 2 shows the spectrograph with the lid removed, seen from above. The plane surface of one long wall is turned outwards. To this wall are attached the tombac tube A for evacuation, the spark-chamber B with yet another tombac tube to the pump, the arrangement C for regulating and the needle-valve D. Another needle-valve E is placed on one of the short walls to be used as a regulating leak for the vacuum.

The spark-chamber — the X-ray tube. The body of the spark-chamber is a parallelepiped of forged brass, which is truncated at an angle of 45°. A strong brass plate is firmly screwed and soldered to the truncated surface. This plate is, in turn, vacuum-tightly screwed to the wall of the spectrograph. This should be clear from fig. 2. A more detailed picture of the spark-chamber is given in fig. 3, which shows a vertical section containing the light-path. Here B is the body of the chamber and A the wall of the tank. A vertical hole is bored through the body B for the electrodes. The radiation from the spark passes through a horizontal aperture which continues through the wall into the tank. The beam must thus pass through the thin aluminium foil C which covers the rectangular opening in the screen D. The height and breadth of the opening in the screen are 3 mm and 12 mm respectively. The tube E, to which the screen is attached, fits exactly into the horizontal aperture in the spark-chamber and enters the spectrograph at an angle of 45° to the wall A (compare fig. 2). Inside and concentric with the tube E is a smaller tube F, into which the two screens G and H may be wedged. In the tube F there are holes for evacuating the space between the tubes. It is possible to make holes in the tube E also, and in this way obtain a totally dark communication between the spark-chamber and the tank. This however did not prove to be necessary. The screen G, which has a rectangular opening of height 4 mm and breadth 12 mm, limits the height of the incident beam, and, together with the screen H, prevents the foil C from being torn by solid particles emanating from the spark. The distance between the foil C and the spark may be varied considerably, since the tubes E and F may be changed for others of different lengths. The open end of the horizontal aperture is closed by a window I. Furthermore, in one of the otherwise unbored surfaces of the chamber a horizontal hole is bored for the evacuation (compare fig. 2). There is also in the body of the chamber a channel for cooling water.

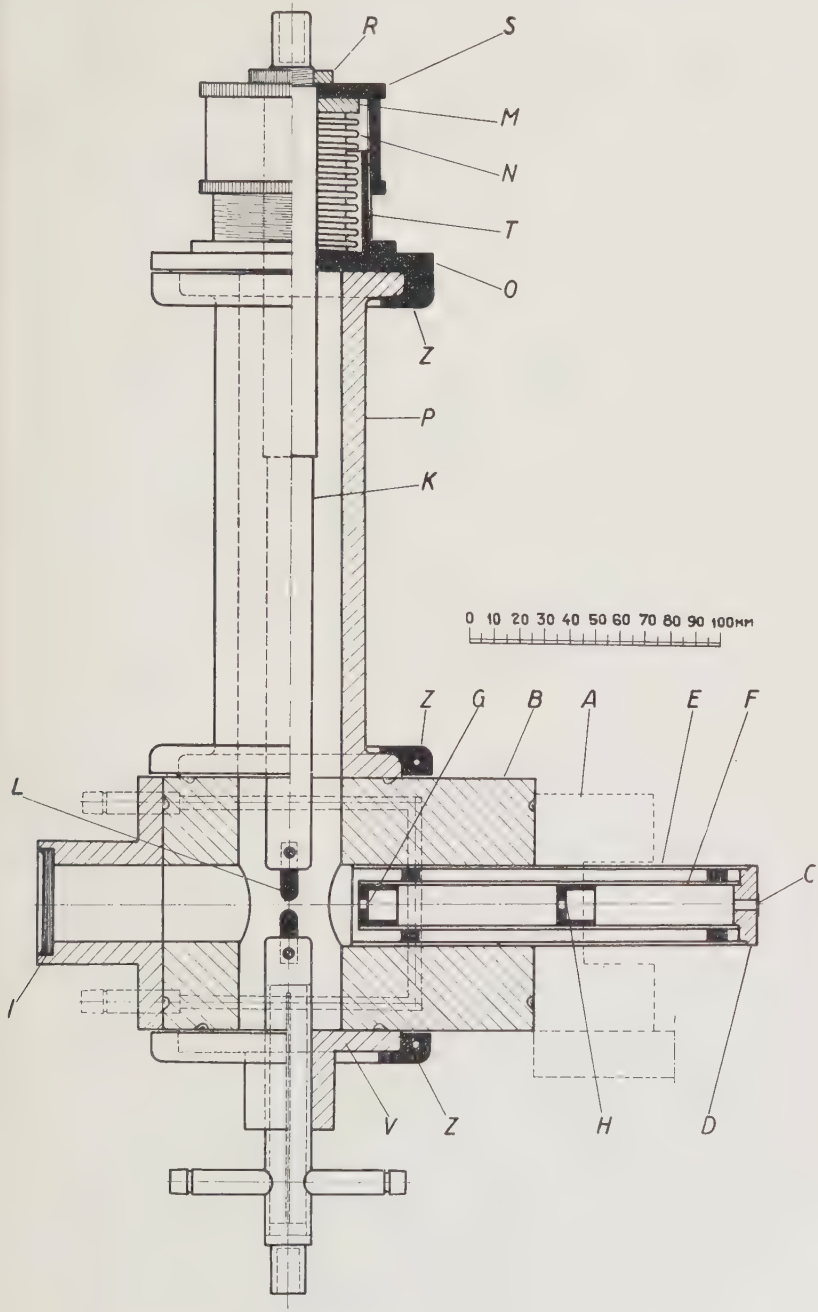

Fig. 3. The spark-chamber.

The high voltage electrode consists of a homogeneous brass rod K with a hole into which the actual electrode L may be tightly screwed. A plate M is soldered to the upper end of the rod. This plate is vacuum-tightly joined, by means of the sylphon bellow N, to the plate O, which rests on the porcelain insulator P. The ring R and the threaded sockets S and T, of which T is firmly attached to the plate O, render

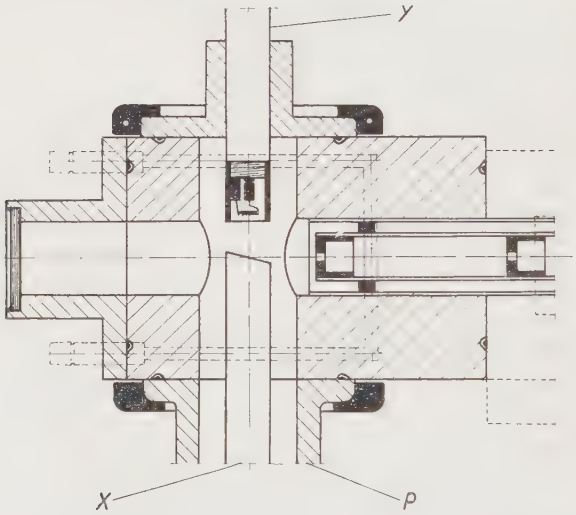

Fig. 4. The chamber mounted as an X-ray tube.

it possible to displace the electrode vertically. The lower electrode, which is earthed, is attached to the body of the chamber by means of the plate V. This electrode is constructed so that it may be water cooled. The flanges of the insulator and the plate V are pressed against the rubber packing by means of the divisible rings Z, which are affixed with sunken screws to the base.

The spark-chamber described above may easily be changed into an X-ray tube. As shown in fig. 4, it is only necessary to exchange the spark-electrodes for X-ray electrodes. A porcelain insulator P, similar to the one used for the upper spark-electrode, insulates the anticathode X from the body of the tube. The anticathode is introduced from below, so as not to disturb the powdered substances rubbed into it. The cathode Y, which is earthed and connected to the body of the tube, is supplied with a low temperature filament. Both X-ray electrodes have channels for cooling water.

The spectroscopic unit. We now reconsider fig. 2. The spectroscopic unit consists of a 10 mm thick sector-shaped base-plate F with radius 250 mm. The plate G, which is firmly attached to the base-plate, carries the crystal-table with the crystal-holder. The sector-shaped arm H, which carries the plate-holder I, may be rotated about the axis K. This axis is movable along a slit in the base-plate, so that the distance between the axis and the mid-point of the crystal may be varied. The construction is so made that the axis may be locked in the desired position, while the arm H may still rotate. The angle subtended by the mid-points of the crystal and the plate at the axis K may be varied from 40° to 138°. The angle is read from the scale L, which is marked off in whole degrees. The scale is turnable about the axis K, and moves with it. The arm H may be locked in the desired position by means of the screw clamps M. The distance from the mid-point of the plate to the axis K may also be varied as the plate-holder is mounted on a sliding plate N. The screw O is used to lock this plate. Thus it is possible to adjust the spectrograph so that the distances from the axis K to the mid-point of the crystal and to the mid-point of the plate correspond to the radius of the given focussing circle. (This radius is not usually exactly equal to half the radius of the crystal-holder's cylindrical surfaces). Once this adjustment has been made, it is only necessary to loosen the screw clamps M in order to readjust for the different glancing angles.

The spectroscopic unit has three steel balls as feet. One of these is directly underneath the mid-point of the crystal, another at P and the third at the edge of the base-plate F half way between the other two. The balls are mounted on the base F so that they can be adjusted vertically. The ball under the crystal can, by means of a ring, be fixed in the centre of one of the five holes R in the plate S. As the other two balls roll on the bottom of the spectrograph, the spectroscopic unit may thus be turned about a vertical axis through the mid-point of the crystal. Five circular scales, with the mid-points of the five holes as centres, are engraved on the plate S. By means of these scales and the projection T as index, the spectroscopic unit can be adjusted for the desired glancing angle. Finally, the vertical plate V is screwed tightly to the base-plate F. To the former plate is attached the jointed rod Z to the regulating arrangement C, which contains a sylphon bellow mounted in the same way as that in the upper spark-electrode. The arrangement C renders it possible — even when the spectrograph is evacuated — to alter, within

certain limits, the angle at which the incident beam strikes the crystal. By loosening the rod Z, the spectroscopic unit may, if so desired, be removed from the body of the spectrograph. The spectroscopic unit consists entirely of brass, except for the crystal-holder, screws, etc.

The plate-holder. A detailed picture of the plate-holder and mount is given in fig. 5, which shows a vertical section in the plane of symmetry and the construction seen from above. The letters F, H, I, M, N and O have the same significance in this figure as in fig. 2. The plate-holder I, which has a rectangular opening 6.5×47 mm^2, is attached to the circular plate A. This may be turned about the vertical axis B, which coincides with the axis of symmetry of the circular plate and the centre line of the photographic plate. It is thus possible

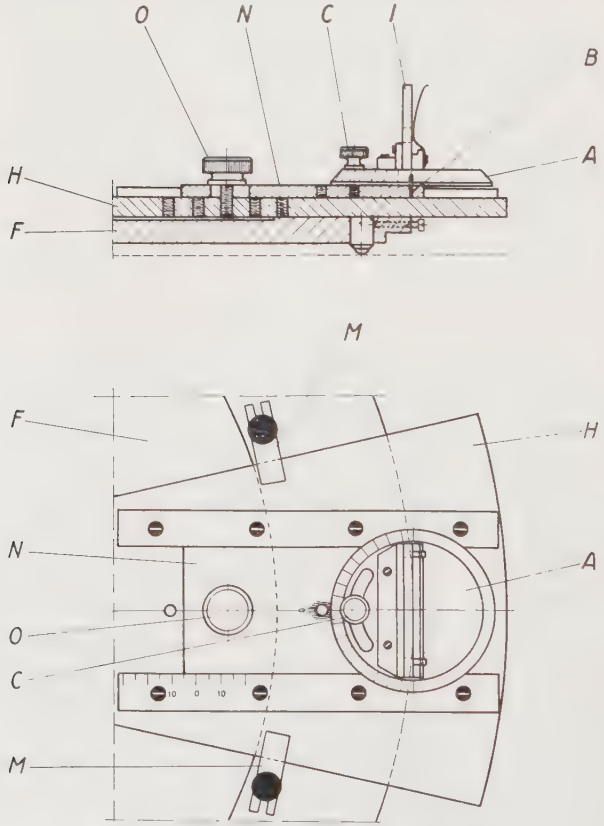

Fig. 5. The plate-holder and its mount.

to make use of different plate positions. The normal at the mid-point of the photographic plate may pass through either the centre of the focussing circle or the mid-point of the crystal. The screw C is used to lock the plate-holder in the desired position. On the vertical section in fig. 5 may also be seen the attachment for one of the steel balls which serve as feet for the spectroscopic unit.

The disc A with the plate-holder may be completely removed, and replaced by a cylindrical plate-holder ground to a radius of 250 mm and intended for film. The film-holder, however, was not used in this investigation.

The crystal-holder and crystal-table. The construction of the crystal-holder is shown in fig. 6. The holder is made of

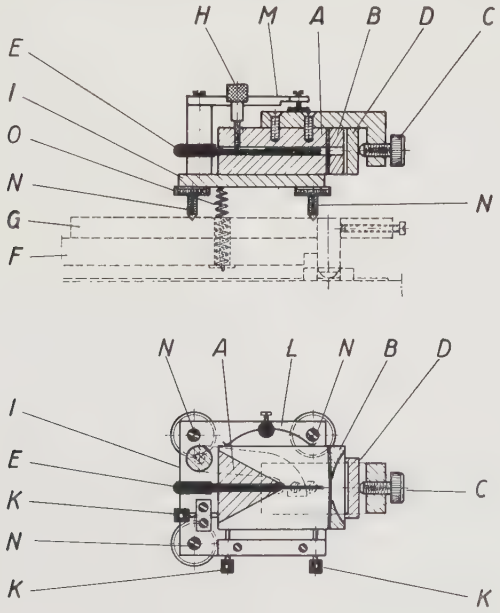

Fig. 6. The crystal-holder and crystal-table.

hardened special steel. The cylindrical surfaces of the main parts A and B have a radius of 500 mm, and are ground and then treated in a special way so that the surfaces are very smooth and the shape exact (compare INGELSTAM (21)). The crystal itself is fixed between the blocks A and B by the screw C and the connecting piece D, which presses with only two edges against the block B. A large, 3.0 mm high, horizontal

aperture in the centre of block A renders it possible for the beam to pass to and from the crystal. There is also an opening of 3.5×13 mm^2 in block B. The active crystal surface may be limited by the shutter E, which is locked by the screw H. This type of crystal-holder has been used at this Institute for several years, but the author's holder is superior to the older ones as regards the quality of the cylindrical surfaces.

The crystal-holder rests with the plane under-surface of the block A on the table I. The adjusting screws K and the spring L fix its position, whereupon it is locked by the arrangement M. When in the correct position, the block B does not touch the table. The height of the table and its inclination to the base may be altered by means of the three screws N. A strong spring O presses the points of the screws against the plate G, which is firmly attached to the base-plate F of the spectroscopic unit. The figure shows also the attachment for the steel ball which lies under the mid-point of the crystal.

This crystal-holder may be used from the smallest possible glancing angles up to somewhat over 60°. For larger angles, there is another holder and table; but these were not necessary in the present investigation.

Fig. 7 shows the spectroscopic unit in position inside the

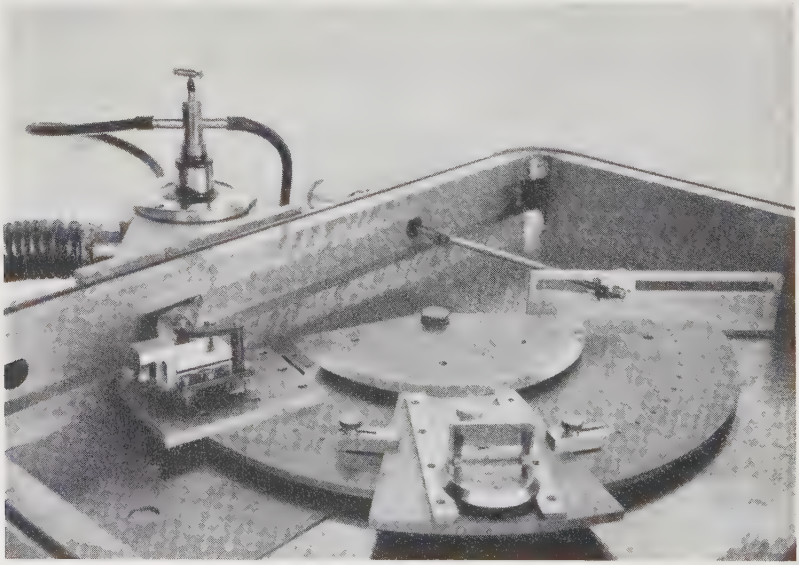

Fig. 7. The spectroscopic unit in position inside the vacuum tank.

vacuum tank. Both the crystal-holder and the plate-holder and their mounts may be seen. The source of radiation is arranged as an X-ray tube, and the electrode shown in the picture is, therefore, the cathode.

The crystals. The crystals used were of mica and gypsum. The reflecting planes were (001) in the case of mica and (010) for gypsum. These are the natural cleavage planes, and as such, are those generally used in X-ray spectroscopy. The mica crystal was cut from a large, very fine sheet supplied by Mica and Micanite Supplies, Ltd., London. It was easy to bend, and a crystal opening of at least 12 mm could be used without causing any noticeable error in the image.

No less than 6 gypsum crystals were used. It became apparent that the reflecting power of such crystals decreases after a period of use, especially for the relatively long-waved optical radiation. At the same time, the surface of the originally perfectly transparent crystal becomes clouded in the region struck by the radiation. A gypsum crystal placed in the spectrograph, but protected against the radiation, did not change, at any rate not in the time necessary for the destruction of the real crystal. The change is certainly connected with a partial disappearance of the water of crystallization, which gypsum normally contains. The phenomenon has already been observed by, for instance, Saur (29). He considered that the disappearance of the water was caused by the high vacuum alone. The present investigation, on the contrary, would indicate that the radiation at least accelerates the process. The discrepancy between these two results may possibly be due to the fact that Saur worked with soft X-rays, while in the present investigation hard X-rays were emitted very often immediately before each vacuum spark. Two gypsum crystals were supplied by the firm of Steeg & Reuter, Bad-Homburg, while the rest were split and cut out from a large piece of Montmartre gypsum. It was difficult to obtain perfect crystals and the largest crystal opening which could be used was, therefore, in general only 7 to 8 mm. A summary of the dimensions and radii of curvature of the crystals is given in table 1. The table also shows which crystal was used for a given plate. Note how few exposures could be made with the same crystal. Considering that it takes several days to make and adjust a new crystal, it is clear that in any future investigations of this kind, gypsum should not be used.

An idea of the inverse linear dispersion is given in table 2, where the number of X. U. per mm of plate is calculated for

Table 1.

Crystal	Size in mm²	Radius in mm	Plate No.
Mica	28×17×0.5	496	321—368
Gypsum I (Montmartre)	21×13×0.3	496	369—378
Gypsum II	24× 9×0.6	492	379—385
Gypsum III	24× 9×0.5	492	386—388
Gypsum IV (Montmartre)	16×15×0.6	486	389—396
Gypsum V (Montmartre)	19×13×0.3	484	397—399
Gypsum VI (Montmartre)	17×10×0.4	494	400

Table 2.

Crystal	Gypsum Radius = 490 mm			Mica Radius = 496 mm		
λ X. U.	6 000	7 500	9 000	13 000	15 000	17 000
X. U. mm	28.4	26.9	24.9	30.3	26.2	20.7

several different wave-lengths, on the assumption that the plate touches the focussing cylinder. The radius of curvature of the gypsum crystal was assumed to be 490 mm.

B. Remaining Experimental Equipment.

The pumps. The vacuum tank of the spectrograph was evacuated by means of a modern molecular pump of the disc type, constructed at the Research Institute for Physics in Stockholm. It is backed by an ordinary rotary oil pump. Two tombac tubes, with an inner diameter of 40 mm, issue from the molecular pump; one to the spark-chamber and the other to the vacuum tank. A valve on the molecular pump renders it possible to stop the pumps without breaking the vacuum in the spectrograph.

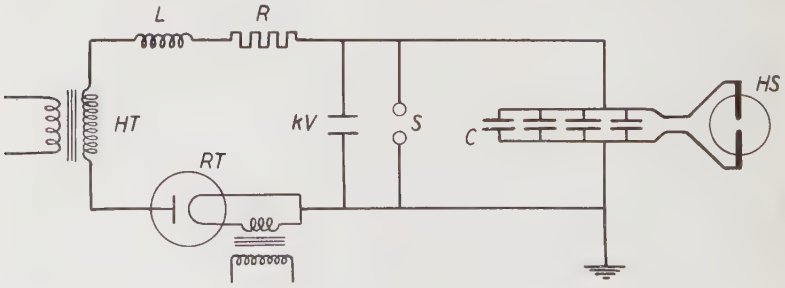

Fig. 8. The high voltage circuit for the vacuum spark.

The high voltage sets. A small high voltage set was used for the X-ray recordings. It consisted mainly of a transformer, two rectifying valves and two condensers in Greinacher coupling. The set gives a maximum voltage of 20 kV, which is ample for obtaining all the reference lines in question.

Fig. 8 shows the coupling of the larger high voltage set, which gives the vacuum spark. The condenser-battery C consists of four condensers in parallel, each with a capacity 0.1 μ F and withstanding a maximum voltage of 80 kV. The condensers are charged by means of the high voltage transformer HT and the rectifying valve RT. The transformer gives a maximum of 100 kV, but was not fully used because the safety spark gap S was adjusted for 75 kV. In order to obtain the highest possible ionization power in the vacuum spark, the wires between the condensers themselves and to the spark-chamber HS were arranged so that self-induction within the discharging circuit should be as small as possible (1). A static voltmeter kV shows the voltage. The induction coil L and the resistance R of 300 000 Ω reduce the current in the transformer when the condensers discharge.

The thermostat. It was in general necessary to expose the plates for so long that the work on each plate took more than

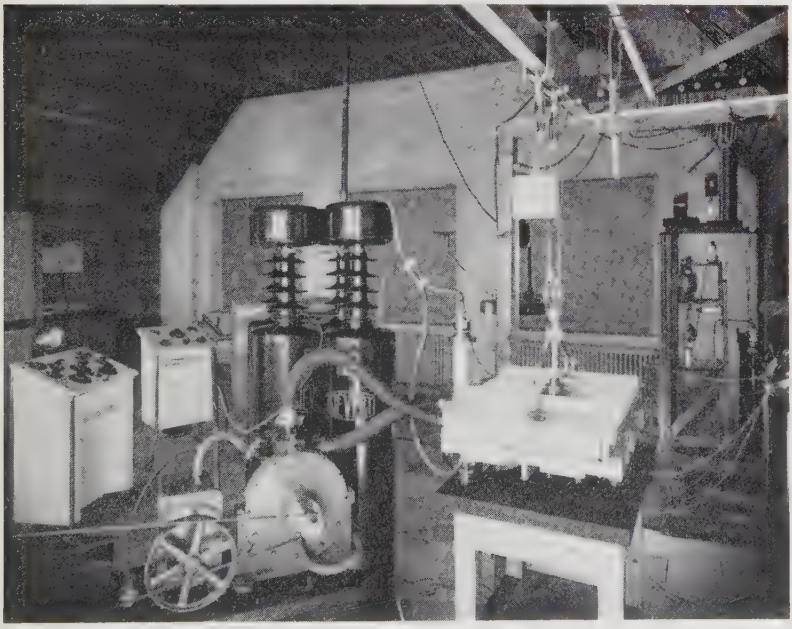

Fig. 9. View of the entire apparatus.

a day. The room temperature was, therefore, kept to within less than one degree of constant by means of a simple thermostat. A thermometer placed above the spectrograph influences a relay which couples or uncouples an electric stove. A fan achieves the necessary circulation of the air.

Fig. 9 shows most of the experimental equipment. The vacuum tank and the pulley and pulley-carriage for removing its lid and the high voltage electrode of the spark-chamber are shown. The smaller high voltage set is on the extreme right, with its switchboard in the background on the left. On the extreme left is the switchboard of the larger high voltage set, the transformer and rectifying valve of which are in a neighbouring room. The condenser-battery of this set may be seen in the centre of the picture. The pumps are in the left foreground. The thermometer belonging to the thermostat hangs near the pulley-carriage.

IV. The Recording and Measurement of the Spectrograms.

Adjustment of the spectrograph. Complete adjustment of the spectrograph is achieved in the following manner:

1) The mid-point of the crystal must lie on the axis of rotation of the spectroscopic unit. This axis is at right angles to the plate G, and its point of intersection with the plate is also known (fig. 6). Faint scratches may be made on the edge of the crystal-table nearest to the axis to indicate the correct position of the crystal-holder relative to the edge.

2) The mid-points of the crystal and the plate must be at the same height above the base of the spectroscopic unit. A surface gauge may be used to test this. The screws N (fig. 6) are turned until the condition is fulfilled.

3) The normal at the mid-point of the crystal must be parallel to the base of the spectroscopic unit, and intersect the axis K (fig. 2) of the unit. The shutter E (fig. 6) is removed. A short slit is placed at right angles to the base at the point on the focussing circle diametrically opposite to the mid-point of the crystal. A surface gauge may be used to check that the bottom of the slit is at the same height above the base as the mid-point of the crystal. A narrow, almost parallel beam issuing from the slit passes through the hole for the shutter E and is reflected by the crystal. Adjustments are made until the image lies in the same straight

line as the slit and immediately below it and, at the same time, the narrow beam is bisected by the axis K. The screws N and K are turned to achieve this.

4) The normal at the mid-point of the plate must both be parallel to the base of the spectroscopic unit, and intersect the axis K (fig. 2) of the unit. The first condition was fulfilled during the construction of the spectrograph. The plate-holder is turned about the axis B (fig. 5) until the other condition is also fulfilled. This may be tested by using a suitable set-square.

5) The base of the spectroscopic unit must be parallel to the plane bottom of the vacuum tank, and the mid-points of the crystal and the opening in the screen D (fig. 3) must lie at the same height above the bottom. This may be tested with a surface gauge. The lengths of the three feet of the spectroscopic unit are adjusted until the conditions are fulfilled.

6) The centre of the focus-spot must be at the same height above the bottom of the vacuum tank as the mid-points of the crystal and the opening in the plate-holder. This may be checked either with a surface gauge or by taking a pinhole camera exposure. The anticathode may be screwed vertically until the focus-spot is in the correct position.

7) The radius of curvature of the crystal, which is known approximately from the optical tests in connection with the bending, is measured more accurately by means of X-ray recordings. For this purpose, the plate is inserted in the plate-holder so as to make only a small angle with the base of the spectroscopic unit (focussing plate). The position of the plate-holder is shown by a mark on the photographic emulsion. The position of the intersection of the focussing cylinder and the plate is indicated because the spectral lines are sharper there than elsewhere. The spectroscopic unit is adjusted for the radius thus obtained by displacing the axis K and the sliding plate N (fig. 2).

8) The generatrices of the cylindrical surface of the crystal must be perpendicular to the base of the spectroscopic unit. This is tested by means of a normal X-ray recording. If the lines on the spectrogram are not at right angles to the longitudinal direction of the spectrogram, the crystal must be turned about the normal at its mid-point. The screws N (fig. 6) are used for this purpose. This X-ray recording also shows whether the height of the focus-spot is correct or not, because the entire length of the spectral lines is only a

little greater than the height of the opening in the plate-holder. The positions of the spectral lines on the plate are also measured and compared with the calculated values. Any discrepancy shows that the normal at the mid-point of the crystal does not intersect the axis K or that the mid-points of the crystal and the plate are not equidistant from the axis K (fig. 2).

Plate material. All the optical spectra were recorded on Ilford Iso-Record plates 4.5×6 cm^2, which had been cut before use to a size 2.25×6 cm^2. The emulsion on the plates was the same as that on the earlier Imperial Eclipse plates. The plates are sensitive, but have not a particularly fine grain. The latter matters little, as the linear dispersion is great. The sensitivity, on the other hand, is of the greatest importance because many optical lines emerge only after long exposure. Certain X-ray recordings for purposes of adjustment were, however, made on the less sensitive but more finely grained Agfa Contrast plates. All the plates were developed with the ordinary metoquinone developer.

Since the optical spectra under consideration lie within the wave-length region 6—17 Å. U., there is certainly nothing to be gained by using Schumann plates. SIEGBAHN and MAGNUSSON (31), moreover, consider that ordinary Eclipse plates are superior to Schumann plates for the entire region 10—50 Å. U.

Spark-electrodes and absorbing foils. The optical spectra measured in this investigation, belong to the elements aluminium, magnesium and fluorine. In the case of aluminium and magnesium, the upper and negative electrode consisted of a cylindrical rod, 6 to 8 mm thick, of the metal in question. The lower electrode, which was earthed, was in both cases a graphite rod, 6 mm in diameter. The intensity of the spectrogram obtained appears to be greater when one metal rod and one graphite rod are used instead of two metal rods as electrodes. A further advantage is that a graphite electrode is consumed less quickly than a metal electrode. In the case of fluorine, the upper electrode was a graphite rod while the lower consisted of a natural fluorite crystal (CaF_2). The crystal was ground to the form of a truncated cone and was then riveted securely into a copper socket. Such a crystal withstands a good many hours of exposure and gives a very intensive fluorine spectrum. The fact that the crystal contains calcium is of no importance here, since lines belonging to the optical spectra of calcium do not appear in the wave-

length region in which the required fluorine lines lie. In some spectra, however, a pair of X-ray lines belonging to calcium (the $K\alpha$-doublet in the fifth order, see Plate I c) may be seen.

In all photographic recordings of X-ray spectra with a crystal, a foil must be inserted in the path of the rays to absorb the visible light from the X-ray tube. From the point of view of the intensity, this is naturally a disadvantage, the results of which begin to be apparent as soon as the wave-length of the radiation is greater than 2 or 3 A. U. The same difficulty arises when recording optical spectra with a crystal. As has already been mentioned, the foil was placed on the screen D (fig. 3). It is made up of two or three layers of aluminium foil of thickness 0.5 μ, one on top of the other. The absorption in such a foil is significant at least in certain wave-length regions, see SIEGBAHN (30) page 121 and HILL (17). Nevertheless, it would be difficult to find a more suitable absorption medium. It is obvious that the foil must among other things be mechanically firm, as the opening in the screen is as large as 3×12 mm^2. The foil must, moreover, be perfectly opaque, since the vacuum spark is also an intensive source of visible light.

Because the reference lines were relatively short-waved X-ray lines, an additional aluminium foil, 10—20 μ thick, was inserted in the path of the rays during the exposures of these lines, in order to lessen the continuous blackening. This foil was attached to a special holder, which was placed in front of the screen D (fig. 3).

The reference lines. High orders of the X-ray lines $K\alpha_1$ and $K\alpha_2$ of the elements 25 Mn to 32 Ge were as a rule used as reference lines. In one case a $K\beta_1$-line had also to be used, namely Cu $K\beta_1{}^{XII}$. The anticathode consisted always of a copper plate, into which the pure metal was rubbed. These reference lines are easy to obtain and have a small natural breadth. Their wave-lengths are very accurately known, and are, as SANNER (28) has recently pointed out, the same whether the pure metal or an oxide is used on the anticathode. The latter is not the case, however, with the line Mn $K\alpha_1$, but this last line is used in only one case. The disadvantage of these reference lines is that their wave-lengths are considerably less than those of the required optical lines. As has been shown earlier on, this may introduce a small systematic error in the wave-length values due to unequal penetration of the photographic emulsion. Long-wave lines, which would be more suitable as reference lines, are,

however, scarcely to be found. The wave-lengths of the reference lines are taken from BEARDEN and SHAW (3), whose measurements were carried out using the pure elements and a double crystal spectrometer.

Finally, the reference lines in two cases consisted of lines belonging to the *L*-series for the elements 50 Sn and 51 Sb. In these cases also the pure metal was rubbed into a copper anticathode. For these lines also it is, moreover, certainly true that the wave-lengths are not influenced by chemical combination. The wave-length values in this case are taken from HAGLUND (16).

Method of exposure. When recording optical spectral lines, the approximate wave-length value must first be extrapolated and the corresponding glancing angle calculated. Suitable reference lines are chosen — if possible, one on either side of the required line — and their glancing angles also calculated. By turning the arm H (fig. 2), the plate-holder is placed in a position corresponding to these glancing angles. The shutter E in front of the crystal (fig. 6) is adjusted so that a suitable area of the crystal surface is used. Consideration must here be given both to the quality of the crystal and to the extent of the broadening of the lines, which always accompanies a given crystal opening. The spectroscopic unit is placed in the vacuum tank so that the steel ball under the crystal is in the centre of one of the holes R (fig. 2). The hole should, if possible, be chosen so that the vacuum spark lies just inside the focussing circle. The spectroscopic unit is turned until the angle between the crystal and the incident ray is the same as the glancing angle calculated for the required line, whereupon the rod Z is locked to the plate V (fig. 2). The spectrograph is then evacuated and the spark-spectrum exposed. The voltage immediately before the dischargings was 50—60 kV; in isolated cases higher. The times of exposure varied from 6 to 16 hours with 8—15 sparks a minute. At regular intervals throughout the exposure, a slight alteration is made in the incident angle by means of the arrangement C (fig. 2). The reason for this is partly to assure that the whole opening of the crystal is used and partly to illuminate a larger angle area, especially in cases when the exposure is intended for several neighbouring lines. The vacuum is then broken and the spark-electrodes exchanged for the X-ray electrodes. The thicker aluminium foil is introduced into the path of the beam. In some cases, a screen is placed at the same time on the plate-holder in such a way as to cover, for example,

the upper half of the opening. Thereupon the spectrograph is again evacuated and the exposure of the reference lines begun. The current in the X-ray tube was 1—8 mA and the voltage 10—16 kV. The total time of exposure for both reference lines varied from 20 to 90 minutes. The regulating arrangement C (fig. 2) allows of a sufficiently great alteration in the angle of incidence for both reference lines to be obtained one after the other without breaking the vacuum. Moreover, the angle of incidence is regularly altered as before during the exposure of each line.

The measurement of the spectrograms. The plates were measured in a projection comparator. The instrument has been tested and described by INGELSTAM (20). He found that the periodic error amounted to $\pm$ 0.0007 mm and that lengths measured by the comparator are throughout 0.043 % too great. This latter error has, in our case, no significance. The periodic error may, as a rule, be completely neglected in comparison with the accidental error in adjustment. Moreover, the greater part of the former error can be eliminated if two measurement series are made, and the plate displaced between them so that the screw takes up different angle positions when adjusted on a given line. The measurements of most of the plates were carried out so that after four or five adjustments on all the lines, the plate was turned so that the part which had previously been on the right was now on the left, whereupon a further four or five adjustments were made. In this way, the error arising from a possible subjective tendency to measure to the right or to the left of the maximum blackening in a broad line is avoided. The two measurement series must be made on different parts of the lines so that errors due to inequalities in the grain of the photographic emulsion are, if possible, eliminated. As a rule, the magnification of the comparator was 9 times.

V. Experimental Results.

Wave-length units. It is obvious that the wave-lengths of the optical lines were obtained in the unit in which those of the reference lines were expressed. Because the reference lines were ordinary X-ray lines, they were measured in the conventional X. U. The wave-lengths of the optical lines were, therefore, also obtained in this unit.

All previous wave-length measurements of optical spectra, however, have been made in Å. U. In order that the results

given here may be compared with earlier measurements, the wave-length values must, therefore, be expressed in A .U. This reduction has been made on the assumption that:

$$1\,000.00 \text{ X. U.} = 1.00200 \text{ Å. U.}$$

This relation between the conventional X-unit and the Ångström-unit has been calculated from three particularly careful determinations. Bäcklin (6), who discovered the discrepancy between the two scales, found by repeating accurate measurements that 1 000.00 X. U. = 1.00199 A. U. His value is here corrected by taking into consideration Haglund's (15) more recent accurate measurement of the wave-length of Al $K\alpha_{1,2}$ in X. U. Bearden (2) obtained 1 000.00 X. U. = 1.00202 Å. U. as the average of a large number of measurements. And lastly Tyrén (32), whose final result was 1 000.00 X. U. = 1.00199 Å. U. The reduction factor used in this investigation is the arithmetic mean of these determinations. The error in the ratio can amount only to some units in the last cipher.

The results. As has already been mentioned in the introduction, the main purpose of this investigation was to examine whether spectrographs with curved crystal could be used for recording those optical spectra which are too short-waved to be advantageously obtained with a concave grating spectrograph. It was first of all necessary, then, to find some such spectra which are actually excited in the vacuum spark. It became apparent that the helium I like spectra of certain lighter elements are particularly suitable in this respect.

The first three lines of the principal series $1s^2\,{}^1S_0 - 1snp\,{}^1P_1$ in the helium I like spectra F VIII, Mg XI and Al XII, were the chief lines which could be recorded. The first line $1s^2\,{}^1S_0 - 1s2p\,{}^1P_1$ is accompanied on the long-wave side by a string of satellites. In the case of magnesium, a considerable number of such lines were recorded. This was without doubt mainly due to the fact that the intensity of the magnesium spectra was, on the average, very good.

The results of the wave-length measurements are collected in tables 3, 4 and 5. Since the satellites could not be identified, they have been denoted in the tables only by the letters a, b, c . . . These marks are intended only to indicate corresponding satellites, and have no further significance. It must be pointed out that the satellite b might be the intercombination line $1s^2\,{}^1S_0 - 1s2p\,{}^3P_1$ in the helium I like spectra. It has been identified as such by, for example, Tyrén (32). We shall return to this question later.

The tables show with what crystal and in which order the

Table 3. Fluorine.

Crystal Order Ref.-lines	Plate	Satellites			F VIII		
		d	c	b	$1s^2\ ^1S_0-1s2p\ ^1P_1$	$1s^2\ ^1S_0-1s3p\ ^1P_1$	$1s^2\ ^1S_0-1s4p\ ^1P_1$
Mica	361	17 079.9	17 055.7				
I	362*	17 080.3	17 055.7				
Zn $K\alpha_1^{XII}$							
Cu $K\alpha_1^{XI}$							
Mica	357			16 913.2			
I	358*			16 912.4			
Cu $K\alpha_2^{XI}$							
Cu $K\alpha_1^{XI}$							
Mica	351				16 768.6		
I	352				16 768.7		
Cu $K\alpha_1^{XI}$	356*				16 769.0		
Cu $K\beta_1^{XII}$							
Mica	340					14 428.2	
I	341					14 429.2	
Mn $K\alpha_1^{VII}$	342*					14 428.7	
Zn $K\alpha_1^{X}$							
Mica	344						13 753.6
I	345						13 752.6
Cu $K\alpha_1^{IX}$	347						13 752.5
Fe $K\alpha_1^{VII}$							
	λ X. U.	17 080.1	17 055.7	16 912.8	16 768.8	14 428.7	13 752.9
	λ Å. U.	17.1143	17.0898	16.9466	16.8023	14.4576	13.7804
	$10^{-3}\cdot\nu$ cm^{-1}	5 843.07	5 851.44	5 900.89	5 951.57	6 916.78	7 256.68

Table

Crystal Order Ref.-lines	Plate	Satellites h	g	f	e	d	c
Gypsum	379*	9 389.12	9 372.79	9 364.97	9 298.41	9 276.45	9 264
I							
Fe $K\alpha_1^V$							
Cu $K\alpha_2^{VI}$							
Gypsum	369				9 297.46	9 276.15	9 264
I	370				9 298.70	9 276.69	9 264
Cu $K\alpha_2^{VI}$	372				9 298.34	9 277.44	9 265
Co $K\alpha_1^V$	373					9 277.08	9 265
Gypsum	371				9 298.17	9 277.08	
I	374				9 298.80	9 276.83	
Mg Sat. c							
Mg Xl $1s^2-1s2p$.							
Gypsum	386						
I	389*						
Ga $K\alpha_1^{VI}$							
Cu $K\alpha_1^V$							
Gypsum	388						
I	390						
Ge $K\alpha_1^{VI}$							
Co $K\alpha_1^{IV}$ . . .							
λ X. U.		9 389.1	9 372.8	9 365.0	9 298.3	9 276.8	9 264
λ Å. U.		9.4079	9.3915	9.3837	9.3169	9.2954	9.283
$10^{-3} \cdot \nu$ cm^{-1}		10 629.4	10 647.9	10 656.8	10 733.2	10 758.0	10 772

recording on each plate was made. Similarly, the reference lines used are given. A detailed account of these lines has already been given. The first of the lowest three numbers in each column is the mean in X. U. of the wave-length values given above it; the next is the same mean reduced to Å. U.; while the last is the wave-number in cm^{-1} multiplied by 10^{-3}. If the plate number is marked with a star, then a reproduction of the plate is given in Plate I or II.

The lattice constants of the crystals in the different orders and the correction for anomalous dispersion of gypsum are taken from SIEGBAHN (30). This correction is, moreover, very slight in the case of the wave-lengths concerned here. No correction curve for the anomalous dispersion of mica is to

agnesium.

		Mg XI			
b	a	$1s^2\ ^1S_0 - 1s2p\ ^1P_1$	$1s^2\ ^1S_0 - 1s3p\ ^1P_1$	$1s^2\ ^1S_0 - 1s4p\ ^1P_1$	$1s^2\ ^1S_0 - 1s5p\ ^1P_1$
211.25	9 172.19	9 149.32			
	9 172.35	9 149.63			
212.02	9 173.24	9 149.84			
211.91	9 173.09	9 150.32			
211.91		9 150.22			
212.03	9 173.17				
211.86	9 173.11				
			7 834.66		
			7 834.60		
				7 458.23	
				7 458.36	7 295.02
9 211.8	9 172.9	9 149.9	7 834.6	7 458.3	7 295.0
9.2303	9.1912	9.1682	7.8503	7.4732	7.3096
0 833.9	10 880.0	10 907.3	12 738.4	13 381.1	13 680.6

be found in the literature. However, it may be seen from the tables that only fluorine spectra with wave-lengths between 13 and 17 Å. U. are recorded with mica in the first order. In this wave-length region a correction, if occuring, is of less importance and may probably be neglected. Compare Haglund (15) on the anomalous dispersion of mica.

No X-ray spectra were recorded on either of the plates 371 and 374 for magnesium. They were measured using the lines Mg XI $1s^2\ ^1S_0 - 1s2p\ ^1P_1$ and the Mg satellite c as reference lines instead of X-ray lines. This was done in order to investigate whether the X-ray line Cu $K\alpha_1^{VI}$, which lies very close to the Mg satellite b (see Plate II a), influences the measurement of the latter line. This does not seem to be the case.

Table 5. Aluminium.

Crystal Order Ref.-lines	Plate	Satellites			Al XII		
		d	c	b	$1s^2\ {}^1S_0-1s2p\ {}^1P_1$	$1s^2\ {}^1S_0-1s3p\ {}^1P_1$	$1s^2\ {}^1S_0-1s4p\ {}^1P_1$
Mica	324				7 741.54		
II	325			7 790.36	7 741.17		
Zn $K\alpha_1^{XI}$	327			7 790.48	7 741.43		
Cu $K\alpha_2^{X}$	328*			7 790.42	7 741.28		
	329		7 830.91	7 791.07			
	330	7 840.76	7 830.61	7 791.00			
	333			7 790.72			
Gypsum	392					6 621.62	
I	393*					6 620.96	
Sb $L\alpha_1^{II}$							
Sb $L\beta_1^{II}$							
Gypsum	397						6 300.98
I	398						6 301.20
Sn $L\beta_2^{II}$							
Cu $K\alpha_1^{IV}$							
	λ X. U.	7 840.8	7 830.8	7 790.7	7 741.4	6 621.3	6 301.1
	λ Å. U.	7.8564	7.8464	7.8063	7.7568	6.6345	6.3137
	$10^{-3}\cdot\nu$ cm^{-1}	12 728.5	12 744.7	12 810.2	12 891.9	15 072.7	15 838.6

Plate 379 in table 4 was intended more especially to record the satellites. The angle between the crystal and the incident beam varied, therefore, only within a range corresponding to the glancing angles of the long-waved satellites. Thus, when reflecting the chief line and the satellites a and b, only part of the crystal opening was used. The result of this is shown as clearly as could be desired, because the wave-length values of these lines differ widely from the other values. The investigation with plate 379 offers an obvious illustration of the discussion on this subject given during the consideration of the sources of error. The wave-length values of satellite b in table 5 seem to form two groups. The cause of this is certainly the same. When plates 324—328 were exposed, the glancing angle varied about the value corresponding to the chief line. In the case of plates 329—333, on the other hand, the spectrograph was adjusted for recording the satellites c and d. Thus, in neither case did the satellite b lie within the specifically illuminated angle region. The mean of the wave-length values for this line ought, however, to be nearly as accurate as the means for neighbouring lines, since the parts of the crystal not entering into the reflection in the two cases lay on opposite sides of the mid-point.

The ground states $1s^2\ ^1S_0$ and terms $1snp\ ^1P_1$ of the helium I like spectra F VIII, Mg XI and Al XII were calculated on the assumption that the P-terms could be expressed by means of Ritz's formula. The value of the ground term was, thereby, adjusted until the quantum defects for the first three P-terms formed a linear function of the term values. In the principal series of Mg XI, the fourth line could also be measured, but only on one plate. Even there, the line was very weak. Since the accuracy of this measurement cannot be very great, the line was not taken into account in the calculation of the limit. The results of the calculations are to be found in tables 6, 7 and 8. The wave-numbers and term values are expressed in cm^{-1}, but multiplied by 10^{-3}. The tables contain also the effective quantum numbers n* and the ionization potentials I. P. The errors given in the absolute values of the ground states were estimated as follows: the maximum error in each wave-length was added to the wave-length, and the new values used to recalculate the limit. When transforming cm^{-1} to electron volts, the factor 1.2385×10^{-4} was used. This may be obtained if $h/e = 1.3787 \times 10^{-17}$ erg. sec./e.s.u. (due to Ohlin (26)) and $c = 2.9978 \times 10^{10}$ cm/sec. (due to von Friesen (14)).

Table 6. F VIII.

$1s^2\,{}^1S_0 = (7\,694.9 \pm 1.2)\cdot 10^3$ cm^{-1}. I. P. = 953.01 ± 0.15 volts.

n	λ Å. U.	$10^{-3}\cdot\nu$ cm^{-1}	$1snp\ {}^1P_1$	n*
2	16.8023	5 951.57	1 743.33	2.0071
3	14.4576	6 916.78	778.12	3.0043
4	13.7804	7 256.68	438.22	4.0033

Table 7. Mg XI.

$1s^2\,{}^1S_0 = (14\,209.2 \pm 2.5)\cdot 10^3$ cm^{-1}. I. P. = 1 759.81 ± 0.31 volts.

n	λ Å. U.	$10^{-3}\cdot\nu$ cm^{-1}	$1snp\ {}^1P_1$	n*
2	9.1682	10 907.3	3 301.9	2.0053
3	7.8503	12 738.4	1 470.8	3.0046
4	7.4732	13 381.1	828.1	4.0043
5	7.3096	13 680.6	528.6	5.0119

Table 8. Al XII.

$1s^2\,{}^1S_0 = (16\,825.0 \pm 3.0)\cdot 10^3$ cm^{-1}. I. P. = 2 083.78 ± 0.37 volts.

n	λ Å. U.	$10^{-3}\cdot\nu$ cm^{-1}	$1snp\ {}^1P_1$	n*
2	7.7568	12 891.9	3 933.1	2.0044
3	6.6345	15 072.7	1 752.3	3.0030
4	6.3137	15 838.6	986.4	4.0025

Most of the lines obtained with good intensity are reproduced in Plates I and II. All the spectrograms given there are three times as large as the original plates. A detailed description should not be necessary, but some comments would not be out of place. Notice the great dispersion in the recordings of the longest-waved fluorine lines. No less than three plates were necessary to record the line $1s^2\,{}^1S_0$—$1s2p\,{}^1P_1$ and the three satellites, although all four lines lie within as small a wave-length region as 16.8—17.1 Å U. Observe, further, that the line $1s^2\,{}^1S_0$—$1s3p\,{}^1P_1$ of Al XII was also obtained with good intensity. The wave-length in this case, however, is only 6.6 Å. U. An interesting observation may also be made concerning the line-breadths. The

long-waved fluorine lines are very broad in comparison with the reference lines, while a line such as Mg XI $1s^2\ ^1S_0$—$1s3p\ ^1P_1$ with wave-length 7.9 Å. U. appears to have about the same breadth as the $K\alpha$-lines. The cause of this is obviously the deficient resolving power of the crystal for radiation with wave-lengths greater than possibly 15 Å. U.

The first attempts to record optical spectra with a crystal included only lines within the wave-length region 15—19 Å. U.; see FLEMBERG (13). The present writer was of the opinion that these lines were the easiest to obtain; this, however, is scarcely the case. In spite of the large ionization potential, it is easier to record lines with somewhat shorter wave-lengths, because it is only with these wave-lengths that the suitability of the method is really apparent.

It was hoped to be able to record some line in the hydrogen like spectra of the elements under consideration. It is well known that the wave-length values of the lines in these spectra can be calculated theoretically in Å. U. with the greatest accuracy. If it were possible to measure them with a concave crystal spectrograph, the resulting wave-lengths would be expressed in X. U. In this way, a new check would be obtained for the value of the ratio between the conventional X-unit and the Ångström-unit. The ionization power of the vacuum spark, however, proved to be insufficient to excite the hydrogen like spectra. In an attempt to record Mg XII $1s\ ^2S$—$2p\ ^2P$ ($\lambda = 8.42$ Å. U.) with a gypsum crystal, no trace of the desired line could be seen on the plate after 9 hours exposure. The result was the same for F IX $1s\ ^2S$—$2p\ ^2P$ ($\lambda = 14.98$ Å. U.). The attempt in this case was made with a mica crystal and the time of exposure was 7 hours.

It should be possible with the author's experimental equipment to investigate further spectra in the isoelectronic sequence Ne I, Na II, — — —, which TYRÉN (34) succeeded in tracing up to Co XVIII. In an attempt to record with a mica crystal the line $2p\ ^1S_0$—$4d\ ^3D_1$ of Fe XVII with wave-length 12.1 Å. U. (the shortest wave-length measured by TYRÉN), the line was obtained fully measurable after a few hours exposure.

VI. Discussion of the Results.

Accuracy of the wave-length values. We shall now attempt to estimate the error in the wave-length measurements. The unhomogeneousness of the material included in this investigation, renders a general estimate very difficult.

Distinction will, as usual, be made between accidental errors and systematic errors, although the boundry between these two types is uncertain and makes the division somewhat arbitrary. The accidental errors will be treated first. They are:

a) Different mean crystal temperatures during the recording of the spark-spectrum and the reference-spectrum. It is obviously impossible to measure the temperature of the crystal directly, but since that of the spectrograph is kept, by means of the thermostat, to within less than one degree of constant, the difference in the mean temperatures amounts to at most 0.5°. From formula (6) on page 9, this introduces an error of 0.0019 % in the wave-length values. The linear coefficient of expansion used to obtain this figure is that of gypsum, 3.78×10^{-5}. With mica, the error would be considerably less.

b) The centres of the focus-spot and the active region of the spark being at unequal heights. It has already been shown in the section »Some sources of error» that this error may be calculated from formula (3) on page 7, and that the error is greatest when one of the sources of light is at the correct height. Since the height of the opening in the screen G (fig. 3) is 4 mm, the maximum error would occur if the difference in height were 1.5 mm. In the worst case, then, the error is 0.0029 mm, corresponding to 0.08 X. U. or 0.0013 % of the wave-length.

c) The error arising from the measurement of the plate. The reason for this error is the difficulty in adjusting in the comparator exactly upon the maximum blackening of the lines. This difficulty arises more especially with optical lines, which are generally broader or more lightly exposed than the reference lines. When estimating the magnitude of the error, care must be taken to avoid being led astray by the similarity of the readings from adjustments on the same part of a given line. If measurements be made on another part of the same spectral line, the new readings often approach a mean value rather different from the first. Inequalities in the grain of the photographic emulsion are, of course, the reason for this. After a critical examination of the measurement series, it was decided that errors in the measurement of the plates introduced an error in the wave-length of a line amounting at best to 0.003 %, and rising for the weakest lines to possibly 0.007 %.

We now proceed to the systematic errors. They are:

A) The error which may arise since the angle $2\varphi_0$ in the interpolation formula is not known exactly. As has already been said, the error in $2\varphi_0$ could not be greater than 0.4°. Excluding a single extreme case (plates 392 and 393), such an error in the angle causes a maximum error in the wave-length of 0.0011 %. In the case of large glancing angles, this error may be completely neglected.

B) Errors in the reference lines. With K-lines, this error would amount only to 0.001—0.002 %, while for L-lines it might reach 0.003 %.

C) The error arising since the incident beam strikes the plate obliquely and the reference lines have greater penetrating power than the optical lines. This error has already been considered in the section »Some sources of error», where it was shown that if the reference lines penetrate the photographic emulsion completely while the optical lines are absorbed on the surface, the error is $\frac{1}{2} d \cot \varphi$, where d is the thickness of the emulsion and φ the glancing angle. The value of d, which varies in Iso-Record plates between 15 and 25 μ, is found by measuring with a micrometer screw. It may also be measured using a strong microscope. The microscope is focussed first on the grains at the surface of the photographic emulsion and then on the bottom-most grains. The difference in height may be read off on a graduated drum and amounts on the average to 15 μ. A likely mean for d would be 20 μ. Table 9, which is calculated on this

Table 9.

The error due to oblique incidence on the plate.

Glancing angle	60°	44°	37°	30°	24°
$\frac{\Delta\lambda}{\lambda}$ %	0.0007	0.0021	0.0035	0.0060	0.010

assumption, gives the percentage error in the wave-lengths for different glancing angles. In the case of the smallest angles, the error may be significant $\left(\frac{\Delta\lambda}{\lambda}\right.$ is proportional to $\left.\cot^2\varphi\right)$. It must be noticed that only five lines, Mg XI

$1s^2\ ^1S_0$—$1snp\ ^1P_1$ with $n = 3$, 4 and 5 and Al XII $1s^2\ ^1S_0$—$1snp\ ^1P_1$ with $n = 3$ and 4, have been recorded with smaller glancing angles than 37°. And these lines certainly penetrate the photographic emulsion to a certain extent. With the smallest glancing angles, moreover, *L*-lines were used as reference lines, and these in their turn would not penetrate the emulsion completely. The greatest errors in table 9 would, thus, in reality be considerably smaller.

This last error may be said to stand alone, because it always tends in the same direction — towards too short wave-lengths. Therefore, the error C) has not been included in the total error, but has been considered to have more the character of a correction, the magnitude of which it is true is rather uncertain.

The probability that all the errors a)—B) would tend in the same direction and at the same time be of maximum magnitude is slight. We may therefore calculate the total error as the square-root of the sum of the squares of the separate errors. This method of calculating is even more legitimate as the errors themselves were estimated very liberally.

If two total errors be formed, for a favourable and for an unfavourable case, we have as a final result the total limits of error:

$$0.005 - 0.009\ \%.$$

To these must be added the error C) with its sign. The magnitude of this error may be found in table 9 with the reservation that the largest errors are certainly exagerated.

With the exception of the two cases which have already been mentioned in the summary of the results, it is unnecessary to take into account any error due to an incomplete illumination of the crystal.

Comparison with earlier measurements and theoretical values. Since the results given here constitute the first measurements of optical spectra with concave crystals, it should be of special interest to compare them with earlier measurements with concave gratings and with the values calculated theoretically. Of course, it is only the longest wave-lengths in this investigation which have been recorded before. Most of the fluorine lines, however, have been measured by TYRÉN (32) with a concave grating spectrograph. A comparison between TYRÉN's values and those obtained here may be made with the aid of table 10, where all the wave-lengths are expressed in Å. U. It may be seen from the table that a fairly constant dif-

Table 10.

	Satellites			F VIII	
	d	c	b	$1s^2\,{}^1S_0 - 1s\,2p\,{}^1P_1$	$1s^2\,{}^1S_0 - 1s\,3p\,{}^1P_1$
Tyrén . . .	17.121[1]	17.093[1]	16.951	16.807	14.458
Flemberg .	17.1143	17.0898	16.9466	16.8023	14.4576

ference exists between corresponding wave-length values, if the line $1s^2\,{}^1S_0 - 1s3p\,{}^1P_1$, which Tyrén could not measure with the same accuracy as the other lines, be not considered. The discrepancy is so large that it is difficult to explain it satisfactorily. If the error which may be introduced by the reduction of X. U. to Å. U. be first neglected, the error in, for example, the present wave-length value of 16.8023 would be at most 0.0010. An equal error might possibly arise from uncertainty in the reduction factor between the wave-length scales. Nevertheless, Tyrén would need to have an error of about 0.003 Å. U., which is rather large, since the lines would have been measured in the second order.

It was feared, therefore, that some source of error in measurements with a concave crystal had been overlooked. The fluorine lines were measured with a mica crystal of unknown composition. It is consequently uncertain that the crystal used had exactly the lattice constant given in Siegbahn's book (30). Nevertheless, no great difference could exist, for it would have been noticed on account of its influence on the glancing angles. A small difference would effect the wave-length measurements only if the relative variation of the lattice constant in the different orders was also different. This last possibility may, however, be the case. Furthermore, it is possible that the lattice constant of a crystal alters with bending. If the crystal behaves as an elastic body, the lattice constant at right angles to the crystal face would decrease regularly from the concave surface inwards. Should this be the case, and the reference lines and optical lines have very different penetrating powers, an error might arise in the measurements.

In order to investigate experimentally whether this hypothetical errors really exist, some recordings were made of

[1] These values are as yet unpublished but have nevertheless been placed at my disposal. Dr. Tyrén has also on many previous occasions allowed me to use his results before publication. I am much indebted to him not only for his kindness but also for the advantage I derived from it.

Table 11.

Crystal Order Ref.-lines	Plate	Mg $K\alpha_{1,2}$ λ X. U.
Mica	365	9 870.00
I	366	9 869.75
Ni $K\alpha_1^{VI}$	367	9 870.26
Fe $K\alpha_1^{V}$		
	Mean:	9 870.0
	Haglund	9 869.7
	Johnson	9 869.9

Mg $K\alpha_{1,2}$ with the same mica crystal as was used for the other measurements. Table 11 contains the results and, for the sake of comparison, two values of the wave-length taken from other investigations. Haglund's (16) value is the result of an accurate measurement with a vacuum tube spectrograph and a mica crystal. Johnson (23) obtained his wave-length value with gypsum and a concave crystal spectrograph. The results of the three determinations agree within the limits of error.

No error of the type just considered was, therefore, apparent at a wave-length of 10 Å. U. It seems very unlikely that such an error of considerable magnitude would appear when the wave-length increases to 17 A. U. It is certain, then, that the discrepancy between Tyrén's wave-length values and those given here is not due to any serious systematic error in the latter results.

The tables immediately following are intended to show to what extent the results of the investigation are consistent with the results of earlier measurements of the homologous spectra. Table 12 gives the square roots of the wave-numbers ν for the first three lines in the principal series of all the helium like spectra, hitherto examined. For each line, the difference $\Delta\sqrt{\nu}$ maintains a fairly constant value throughout the entire isoelectronic sequence. The new $\Delta\sqrt{\nu}$-values are no exception to this. Table 13 shows how surprisingly well the values of the terms $1s2p\,{}^1P_1$ follows Moseley's law. And this also is true of the term-values determined here. It is obvious that the value of the term $1s2p\,{}^1P_1$ may be accurately extrapolated for one or two further spectra in the sequence. Finally, table 14 gives a summary of the effective quantum numbers for the first three ${}^1P$-terms of all the spectra. Here also, the

Table 12.

Reference	Spectrum	$1s^2\ {}^1S_0 - 1s2p\ {}^1P_1$			$1s^2\ {}^1S_0 - 1s3p\ {}^1P_1$			$1s^2\ {}^1S_0 - 1s4p\ {}^1P_1$		
		λ Å. U.	$\sqrt{\nu}$ $cm^{-\frac{1}{2}}$	$\Delta\sqrt{\nu}$	λ A. U.	$\sqrt{\nu}$ $cm^{-\frac{1}{2}}$	$\Delta\sqrt{\nu}$	λ Å. U.	$\sqrt{\nu}$ $cm^{-\frac{1}{2}}$	$\Delta\sqrt{\nu}$
Boyce a. Robinson (5)	He I	584.331	413.69		537.024	431.52		522.208	437.60	
				294.69			317.98			325.84
Robinson (27) . . .	Li II	199.280	708.38		178.014	749.50		171.575	763.44	
				290.35			314.61			322.76
» . . .	Be III	100.254	998.73		88.314	1 064.11		84.758	1 086.20	
				288.91			313.64			321.90
Tyrén (32)	B IV	60.313	1 287.64		52.682	1 377.75		50.435	1 408.10	
				288.19			313.21			321.55
»	C V	40.270	1 575.83		34.973	1 690.96		33.426	1 729.65	
				287.98			313.13			321.40
»	N VI	28.787	1 863.81		24.898	2 004.09		23.771	2 051.05	
				287.75			312.92			321.31
»	O VII	21.602	2 151.56		18.627	2 317.01		17.768	2 372.36	
				287.68			312.93			
»	F VIII	16.807	2 439.24	288.02	14.458	2 629.94	312.97			321.46
	F VIII	16.8023	2 439.58		14.4576	2 629.98		13.7804	2 693.82	
	Ne IX									
	Na X			3×287.68			3×313.04			3×321.40
	Mg XI	9.1682	3 302.62		7.8503	3 569.09		7.4732	3 658.03	
				287.91			313.27			321.74
	Al XII	7.7568	3 590.53		6.6345	3 882.36		6.3137	3 979.77	

Table 13.

Reference	Spectrum	$1s2p\ ^1P_1$		
		ν cm^{-1}	$\sqrt{\nu}$	$\Delta\sqrt{\nu}$
Robinson (27) . .	He I	27 176	164.85	
				164.18
» . .	Li II	108 263	329.03	
				164.69
» . .	Be III	243 759	493.72	
				165.02
» . .	B IV	433 943	658.74	
				165.40
» . .	C V	679 212	824.14	
				165.31
Tyrén (32)	N VI	979 010	989.45	
				165.45
» . . .	O VII	1 333 800	1 154.90	
				165.52
»	F VIII	1 743 500	1 320.42	165.45
	F VIII	1 743 330	1 320.35	
	Ne IX			
	Na X			3×165.59
	Mg XI	3 301 900	1 817.11	
				166.09
	Al XII	3 933 100	1 983.20	

Table 14.

Effective quantum numbers of the ^{1}P-terms.

Reference	Spectrum	n*		
Edlén (8)	He I	2.0094	3.0111	4.0115
»	Li II	2.0135	3.0136	4.0136
Robinson (27) . .	Be III	2.0128	3.0114	4.0107
Tyrén (32)	B IV	2.0115	3.0100	4.0096
»	C V	2.0097	3.0085	4.0081
»	N VI	2.0088	3.0086	4.0074
»	O VII	2.0078	3.0075	4.0069
»	F VIII	2.0070	3.0068	
	F VIII	2.0071	3.0043	4.0033
	Ne IX			
	Na X			
	Mg XI	2.0053	3.0046	4.0043
	Al XII	2.0044	3.0030	4.0025

new values continue very well the sequence formed by the earlier ones. The fact that the quantum defect within each spectrum varies somewhat more for F VIII, Mg XI and Al XII than for the other spectra, is due chiefly to the method

used here to determine the values of the ground states (see section »The results»).

As is well known, it is possible to calculate theoretically the value of the ground state $1s^2\,{}^1S_0$ for the helium I like spectra. HYLLERAAS (19) succeeded in deriving a general formula according to which the value of the ground state for an arbitrary spectrum within the sequence could be found. His formula has since been somewhat modified by ROBINSON (27). The values in column 2 of table 15 were obtained by means

Table 15.

Calculated and experimental value of the ground state $1s^2\,{}^1S_0$ in He-like spectra.

Spectrum	Hylleraas' mod. formula cm^{-1}	Relativity correct. cm^{-1}	Mass pol. corr. cm^{-1}	ν cm^{-1} calculated	ν cm^{-1} experimental	Difference calc.-exp.
He I	198 322	2	−5	198 319	198 305 ± 15	+ 14
Li II	610 058	38	−4	610 092	610 079 ± 25	+ 13
Be III	1 241 151	161	−3	1 241 309	1 241 225 ± 100	+ 84
B IV	2 091 672	484	−1	2 092 155	2 091 960 ± 200	+ 195
C V	3 161 626	1 135	0	3 162 761	3 162 450 ± 300	+ 311
N VI	4 451 052	2 284	+1	4 453 337	4 452 800 ± 500	+ 537
O VII	5 959 945	4 133	+2	5 964 080	5 963 000 ± 600	+1 080
F VIII	7 688 318	6 923	+3	7 695 244	7 693 400 ± 800	+1 844
F VIII	7 688 318	6 923	+3	7 695 244	7 694 900 ± 1 200	+ 344
Ne IX	9 636 130	10 927	+4	9 647 061	—	
Na X	11 803 444	16 455	+5	11 819 904	—	
Mg XI	14 190 202	23 853	+6	14 214 061	14 209 200 ± 2 500	+4 861
Al XII	16 796 463	33 497	+6	16 829 966	16 825 000 ± 3 000	+4 966

of the later formula. To these values must then be added a relativity correction for which a general formula has been worked out by ERIKSSON (12). In order to obtain this correction in cm^{-1}, it must be multiplied by the square of the fine-structure constant α. The values in the table were calculated with $\alpha = 7.302 \times 10^{-3}$. Finally, must be added an unimportant mass polarization correction calculated by ROBINSON (27). The final theoretical value of the ground state may be found in the fifth column of the table. The next column gives the values determined experimentally. For He I—Be III, the term-values and limits of error are taken from ROBINSON (27), while the same quantities for B IV—F VIII are from TYRÉN (32). Finally, the results of the present investigation are also given. The last column contains the differences between

the theoretical and experimental values. As TYRÉN has already pointed out, the theoretical values are throughout greater than the experimental ones. This discrepancy is, for the last elements, even greater than the limits of error; in spite of the fact that these limits were chosen rather large.

The satellites. In connection with the summary of the results of the wave-length measurements, a number of lines were mentioned under the general heading of satellites. Thus, in the case of Mg XI the resonance line $1s^2\ ^1S_0—1s2p\ ^1P_1$ was followed on the long-wave side by a considerable number of such lines, of which no less than eight could be measured (see table 4). On one of the original plates, it is possible to see a further two or three lines with longer wave-lengths than the satellite h, among them a line of about 9.50 Å. U. Most of the Mg-satellites measured are shown in the reproduction in Plate II a. Satellite b was easiest to obtain as its intensity approached that of the chief line, and after b, satellites c and d. Since the spectrograms obtained of fluorine and aluminium were less intensive, it was possible with these elements to measure the satellites b, c and d only.

The origins of the lines must be transitions of essentially the same type as the resonance line $1s^2—1s2p$: possibly $1s^2 \cdot 2s—1s2p \cdot 2s$, $1s^2 \cdot 2p—1s2p \cdot 2p$ and $1s^2 \cdot 3s—1s2p \cdot 3s$ and other similar transitions in a three-electron spectrum. Certain satellites might correspond to analogous transitions in a four-electron spectrum. EDLÉN and TYRÉN, who observed and measured a large number of lines of this kind during their investigations of the spectra of the elements boron to fluorine, were the first to suggest this interpretation (10). The values of the terms included in the transitions suggested above have been calculated in a wave mechanical way by WU (36) for all the atoms from lithium to fluorine. The wave-length values thus obtained for such transitions in C IV agree well with those found experimentally for the strongest satellites of the resonance line of C V. Unfortunately, the accuracy of the theoretical values is not sufficient for an exact identification of the individual lines. The experimental results given here should be in a favourable position for comparison with the theoretical ones, because WU pointed out that the accuracy of his calculated energies increases with rising atomic numbers. In any case, the calculations show that the explanation given for the satellites must, in principle, be correct.

EDLÉN and TYRÉN also pointed out in their work quoted above that a line of the type just considered occupies, as regards its wave-length and configuration, an intermediate

position between a true optical line and the corresponding X-ray line. Considering the particular case of magnesium, the resonance line corresponds to the transition $1s^2-1s2p$ while the satellites correspond to longer-waved transitions of the type $1s^2\cdot 2s-1s2p\cdot 2s$, or perhaps $1s^2\cdot 2s^2-1s2p\cdot 2s^2$. The X-ray line Mg $K\alpha_{1,2}$ for a free atom corresponds to the transition $1s^2\cdot 2s^22p^53s^2-1s2p\cdot 2s^22p^53s^2$ and is thus intrinsically the same transition but with the difference that the outer electron shell is added. Compared with the resonance line and its satellites, this introduces a further displacement towards longer wave-lengths. The spectrum b in Plate II illustrates this very well. The upper half of the spectrum was obtained with a vacuum spark and contains the resonance line $1s^2\ {}^1S_0-1s2p\ {}^1P_1$ of Mg XI together with satellites (which thus belong to Mg X or Mg IX). The lower half of the same spectrum was obtained through an X-ray recording with a magnesium anticathode, and contains among other lines Mg $K\alpha_{1,2}$ and its satellites. These X-ray satellites are transitions of the same type as Mg $K\alpha_{1,2}$, although with the former some electrons are missing in the outer electron levels. As the outer electron levels are completed, the corresponding line is displaced from the neighbourhood of the resonance line towards the X-ray line $K\alpha_{1,2}$.

Finally, it must be remembered that the line called here »satellite b» was identified by Edlén and Tyrén as an intercombination $1s^2\ {}^1S_0-1s2p\ {}^3P_1$ in the helium like spectra. However, Wu estimated the intensity of this intercombination line and found that it is very small compared with that of the chief line. Wu considered, therefore, that even this line had the same origin as the other satellites. For this reason, we have so far only called it satellite b and refrained from supporting either side in the question.

Summary.

1) Curved crystals are shown to be suitable for recording optical spectra with wave-lengths of less than 15 Å. U.

2) Some sources of error inherent or occurring in the concave crystal method are discussed. At the same time, factors which must be particularly noted, if the method is to be used for recording optical spectra, are pointed out.

3) A detailed description is given of a vacuum spectrograph with curved crystal, designed by the author and intended for recording optical spectra.

4) The results of some wave-length measurements with the spectrograph of the principal series of the helium I like spectra F VIII, Mg XI and Al XII are given. A comparison is made with earlier measurements of F VIII and the preceding homologous spectra carried out with ruled gratings.

5) The wave-lengths determined are used to calculate the ground state $1s^2\,{}^1S_0$ for the helium like spectra included in this investigation. The values thus obtained are compared with the theoretical values.

6) An account is given of a string of satellites which follow the resonance lines in the spectra measured here.

7) The spectral lines measured lie within the wave-length region 17—6.3 Å. U. Since the shortest wave-length measured with a ruled grating is 12.1 Å. U., the known region of optical spectra has, thus, been extended by about an octave.

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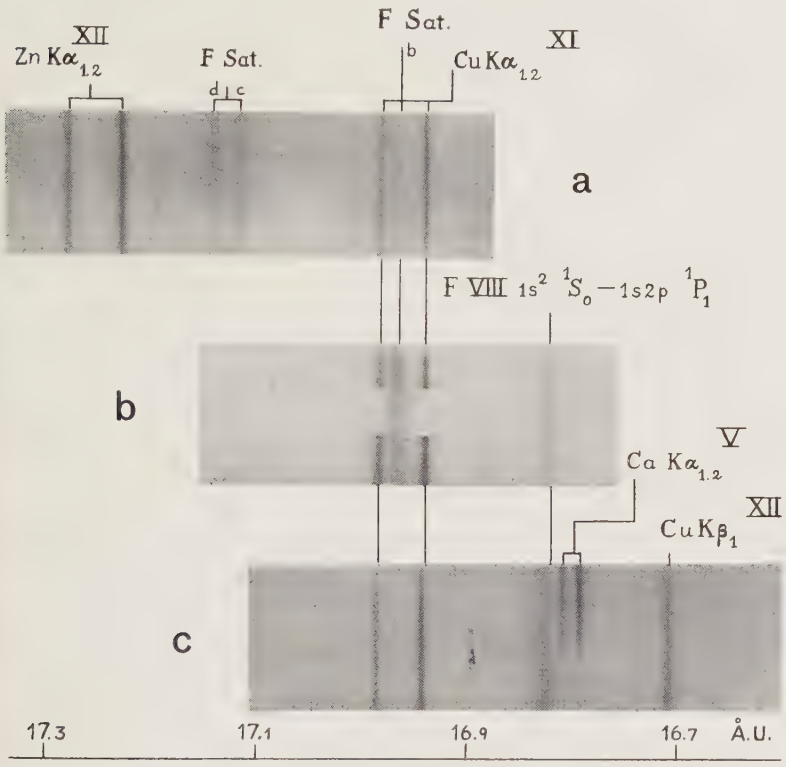
Tryckt den 2 maj 1942.

Uppsala 1942. Almqvist & Wiksells Boktryckeri-A.-B.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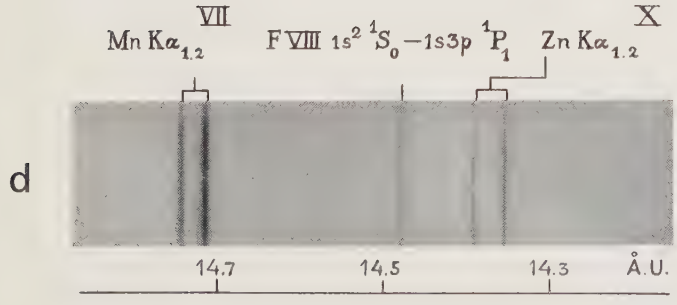
F Sat.
d
c
F Sat.
b
Cu Kα1,2
XI
a
F VIII 1s² ¹S₀ – 1s2p ¹P₁
b
V
Ca Kα1,2
XII
Cu Kβ1
c
17.3
17.1
16.9
16.7
Å.U.


VII
Mn Kα1,2
F VIII 1s² ¹S₀ – 1s3p ¹P₁
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d
14.7
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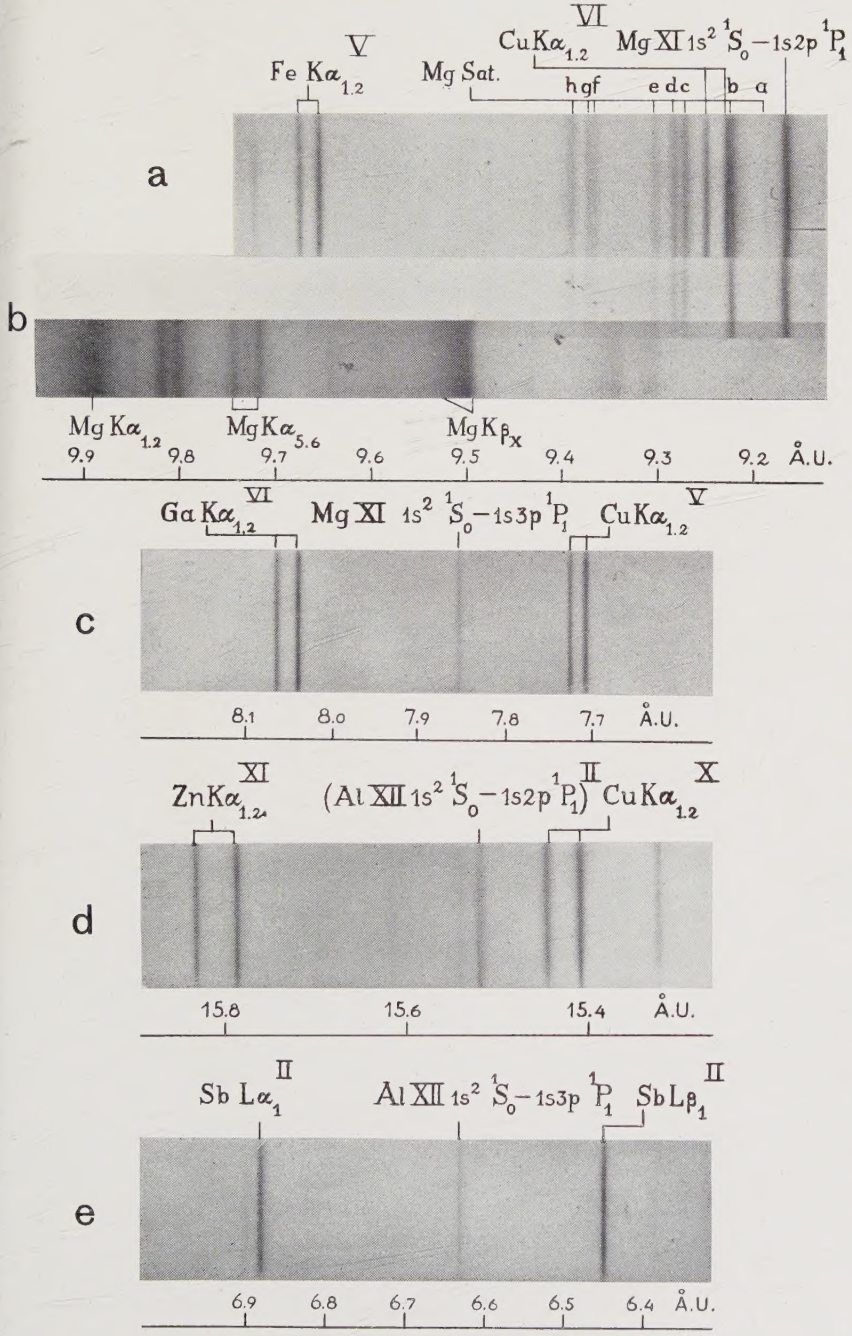
14.3
Å.U.


a
b
c
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e
V
Fe Kα1.2
VI
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MgXI 1s² ¹S₀ – 1s2p ¹P₁
Mg Sat.
h g f
e d c
b a
MgKα1.2
MgKα5.6
MgKβx
9.9 9.8 9.7 9.6 9.5 9.4 9.3 9.2 Å.U.
VI
GaKα1.2
MgXI 1s² ¹S₀ – 1s3p ¹P₁
V
CuKα1.2
8.1 8.0 7.9 7.8 7.7 Å.U.
XI
ZnKα1.2
(AlXIII 1s² ¹S₀ – 1s2p ¹P₁)
II
X
CuKα1.2
15.8 15.6 15.4 Å.U.
II
Sb Lα1
AlXIII 1s² ¹S₀ – 1s3p ¹P₁
II
SbLβ1
6.9 6.8 6.7 6.6 6.5 6.4 Å.U.

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